Chapter 1

Nuclear Fission II

"Thermal neutrons fission ^{235}U *because the compound nucleus is born above the barrier;* ^{238}U *needs fast neutrons because it is born below."*
standard reactor-physics summary [6, 1]

Chapter **??** established that heavy nuclei have $Q > 0$ for fission yet are protected by a barrier of several MeV. This chapter applies that picture to neutron-induced fission of uranium: why thermal neutrons fission ^{235}U but not ^{238}U, what the mass-yield curve looks like, how asymmetry reduces Q relative to the symmetric maximum, why prompt and delayed neutrons appear, how the fission energy is partitioned, and how a chain reaction can be sustained [6, 1, 14, 2].

1.1 Neutron capture on ^{235}U: fissile behaviour

Natural uranium is mostly ^{238}U ($\sim 99.3\%$) with ^{235}U ($\sim 0.7\%$). Thermal-neutron fission of the latter is the workhorse of many reactors. The capture reaction is

$$^{235}\mathrm{U} + n \ \rightarrow \ ^{236}\mathrm{U}^{*} \ \rightarrow \ \text{fission fragments} + \text{neutrons}. \tag{1.1}$$

The capture Q-value (excitation energy deposited in ^{236}U when the neutron has negligible kinetic energy) is

$$Q = [m_n + M(^{235}\mathrm{U}) - M(^{236}\mathrm{U})]c^2 \approx 6.6\,\mathrm{MeV}. \tag{1.2}$$

Thus thermal capture forms ^{236}U* at about 6.6 MeV above its ground state.

Bohr and Wheeler's liquid-drop estimate of the fission barrier in ^{236}U is

$$E_f(^{236}\mathrm{U}) \approx 6.2\,\mathrm{MeV}. \tag{1.3}$$

Because

$$Q \approx 6.6\,\mathrm{MeV} \ > \ E_f \approx 6.2\,\mathrm{MeV}, \tag{1.4}$$

the compound nucleus is formed *above* the barrier. Fission can proceed without tunneling; the fission cross section for thermal neutrons is large (hundreds of barns). Materials that fission with thermal neutrons are called **fissile**; examples are ^{235}U, ^{239}Pu, and ^{233}U [6, 1, 2].

Key idea

Thermal $n + {}^{235}\text{U} \to {}^{236}\text{U}^*$: $Q \sim 6.6\,\text{MeV}$ exceeds $E_f \sim 6.2\,\text{MeV}$. The compound nucleus is born above the barrier: ^{235}U is fissile.

1.1.1 Competing channels

Not every neutron that meets ^{235}U causes fission. Important competitors include:

(i) **Elastic scattering:** $n + {}^{235}\text{U} \to n' + {}^{235}\text{U}$.

(ii) **Radiative capture:** $n + {}^{235}\text{U} \to {}^{236}\text{U}^* \to {}^{236}\text{U} + \gamma$. If enough γ energy is radiated before fission, the excitation drops below E_f and fission becomes improbable. Ground-state ^{236}U then lives for α decay with a multi-million-year half-life.

The branching between fission and (n, γ) is encoded in the fission cross section σ_f and the capture cross section σ_γ [6, 1].

1.2 Neutron capture on ^{238}U: fertile behaviour

For ^{238}U,

$$ {}^{238}\text{U} + n \;\to\; {}^{239}\text{U}^*, \tag{1.5}$$

with capture Q-value

$$ Q = [m_n + M({}^{238}\text{U}) - M({}^{239}\text{U})]c^2 \approx 4.8\,\text{MeV}. \tag{1.6}$$

The fission barrier of ^{239}U remains of order $E_f \approx 6.2\,\text{MeV}$. Now

$$ Q \approx 4.8\,\text{MeV} \;<\; E_f \approx 6.2\,\text{MeV}: \tag{1.7}$$

thermal capture leaves the compound nucleus *below* the barrier. Fission requires tunneling through a residual barrier of order 1.4 MeV, so the thermal fission cross section is negligible.

If the neutron brings kinetic energy K_n, the excitation of ${}^{239}\text{U}^*$ is approximately $Q + K_n$ (neglecting the small centre-of-mass correction). Fission becomes probable when

$$ Q + K_n \gtrsim E_f \quad \Rightarrow \quad K_n \gtrsim E_f - Q \sim 1.4\,\text{MeV}. \tag{1.8}$$

In practice a threshold near 1 MeV is observed: fast neutrons can fission ^{238}U, thermal neutrons cannot. ^{238}U is therefore called **fissionable** (by fast neutrons) but not fissile; it is also **fertile** because successive captures and β decays can breed ^{239}Pu [6, 1, 14].

Caution

The difference between ^{235}U and ^{238}U is not a gross difference in barrier height; both compound barriers are $\sim 6\,\text{MeV}$. The decisive quantity is the neutron separation energy (capture Q) of the compound system: pairing makes even–even ^{236}U more tightly bound relative to ${}^{235}\text{U} + n$ than odd ^{239}U is relative to ${}^{238}\text{U} + n$.

Remark

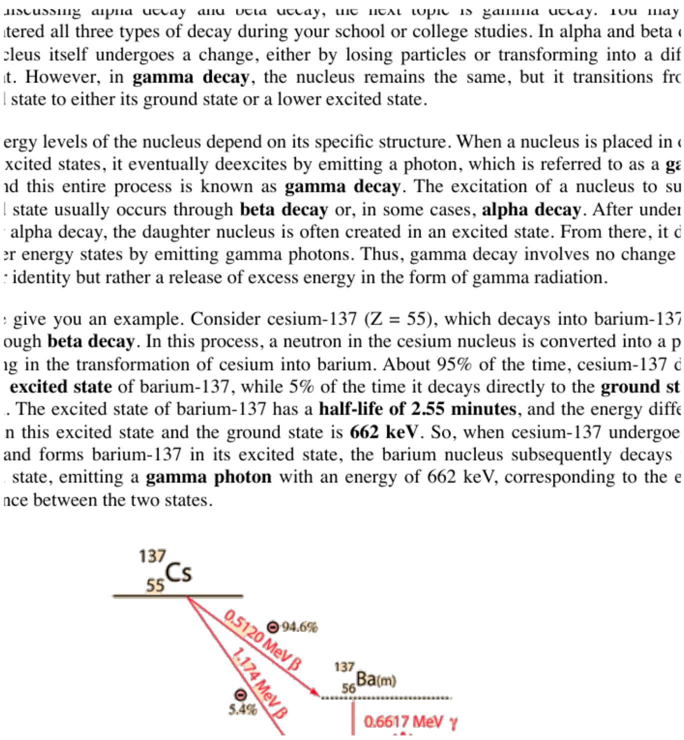
Odd-A actinide targets (^{235}U, ^{239}Pu, ^{233}U) typically have larger capture Q values into even–even compound nuclei and are fissile. Even–even targets more often require fast neutrons [6, 1].

1.3 Fission fragment yields: asymmetric mass distribution

Once $^{236}\mathrm{U}^*$ is above the barrier, it does not split into a unique pair of fragments. Many partitions are open. Representative channels include

$$^{236}\mathrm{U} \to {}^{140}\mathrm{Xe} + {}^{94}\mathrm{Sr} + 2n, \tag{1.9}$$
$$^{236}\mathrm{U} \to {}^{139}\mathrm{Ba} + {}^{94}\mathrm{Kr} + 3n, \tag{1.10}$$
$$^{236}\mathrm{U} \to {}^{141}\mathrm{Cs} + {}^{93}\mathrm{Rb} + 2n, \tag{1.11}$$
$$^{236}\mathrm{U} \to {}^{137}\mathrm{I} + {}^{97}\mathrm{Y} + 2n. \tag{1.12}$$

Plotting the percentage yield against fragment mass number A produces the famous **double-humped mass-yield curve**: peaks near $A \sim 95$ and $A \sim 140$, with a deep valley at symmetric split $A \sim 118$. Symmetric fission is suppressed by roughly two orders of magnitude relative to the asymmetric peaks for thermal fission of ^{235}U [6, 1, 2].


discussing alpha decay and beta decay, the next topic is gamma decay. You may
tered all three types of decay during your school or college studies. In alpha and beta
cleus itself undergoes a change, either by losing particles or transforming into a dif
t. However, in **gamma decay**, the nucleus remains the same, but it transitions fro
state to either its ground state or a lower excited state.

ergy levels of the nucleus depend on its specific structure. When a nucleus is placed in
xcited states, it eventually deexcites by emitting a photon, which is referred to as a **ga**
d this entire process is known as **gamma decay**. The excitation of a nucleus to su
state usually occurs through **beta decay** or, in some cases, **alpha decay**. After under
alpha decay, the daughter nucleus is often created in an excited state. From there, it d
er energy states by emitting gamma photons. Thus, gamma decay involves no change
identity but rather a release of excess energy in the form of gamma radiation.

give you an example. Consider cesium-137 (Z = 55), which decays into barium-137
ough **beta decay**. In this process, a neutron in the cesium nucleus is converted into a p
g in the transformation of cesium into barium. About 95% of the time, cesium-137 d
excited state of barium-137, while 5% of the time it decays directly to the **ground st**
. The excited state of barium-137 has a **half-life of 2.55 minutes**, and the energy diffe
n this excited state and the ground state is **662 keV**. So, when cesium-137 undergoe
and forms barium-137 in its excited state, the barium nucleus subsequently decays
state, emitting a **gamma photon** with an energy of 662 keV, corresponding to the e
nce between the two states.

Figure 1.1: Schematic fission mass-yield curves for fissile nuclei such as ^{233}U, ^{235}U, and ^{239}Pu: asymmetric double peak near $A \sim 95$ and $A \sim 140$. Re-extracted from PH5001 lecture material (Prof. Bharat Kumar).

The pure liquid drop, with only surface and Coulomb energies, tends to favour symmetric scission near the barrier. The observed asymmetry is a **shell-structure** effect: fragments near closed shells (notably near $A \sim 132$, related to the doubly magic ^{132}Sn region) are preferentially formed. Liquid-drop theory remains excellent for the overall energetics and the existence of a barrier; it is incomplete for fine details of the yield curve [6, 14].

1.4 Energy release: B/A curve and asymmetry

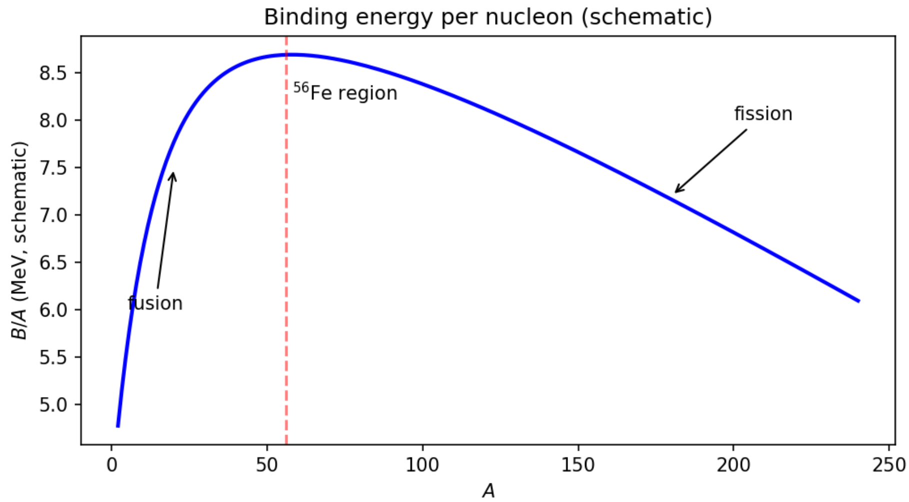


Figure 1.2: Binding energy per nucleon versus mass number. Heavy nuclei sit on the right-hand declining slope; fission products move toward the iron-peak region of higher B/A. Re-extracted from PH5001 lecture material.

Let the parent have mass number A and binding energy per nucleon E_0. Write the fragments as

$$A_1 = \frac{A}{2} - a, \qquad A_2 = \frac{A}{2} + a, \tag{1.13}$$

where a measures the mass asymmetry ($a = 0$ is symmetric fission). Near the fragment region of the B/A curve, assign

$$(B/A)_1 = E_1 + \delta, \qquad (B/A)_2 = E_1 - \delta, \tag{1.14}$$

with δ a small slope correction. The total binding of the fragments is

$$\begin{aligned} E_{B,\text{frag}} &= (E_1 + \delta)\left(\frac{A}{2} - a\right) + (E_1 - \delta)\left(\frac{A}{2} + a\right) \\ &= E_1 A - 2\delta a. \end{aligned} \tag{1.15}$$

The parent binding is $E_{B,\text{parent}} = E_0 A$. Therefore

$$Q = E_{B,\text{frag}} - E_{B,\text{parent}} = (E_1 - E_0)A - 2\delta a. \tag{1.16}$$

Symmetric case ($a = 0$).

$$Q_{\text{sym}} = (E_1 - E_0)A. \tag{1.17}$$

With $E_0 \approx 7.6\,\text{MeV}$, $E_1 \approx 8.5\,\text{MeV}$, and $A \approx 240$,

$$Q_{\text{sym}} \approx 0.9 \times 240 = 216\,\text{MeV}. \tag{1.18}$$

Asymmetric case ($a \neq 0$). The term $-2\delta a$ *reduces* Q. Maximum energy release is for symmetric partition; nature still prefers asymmetric splits because the *barrier* and level densities (shell effects) favour those paths, even though the final Q is slightly smaller [6, 1, 14].

Key idea

$Q = (E_1 - E_0)A - 2\delta a$. Symmetric fission maximises energy release; observed thermal fission of ^{235}U is asymmetric (shell-driven) with Q still of order 200 MeV.

1.5 Prompt neutrons, delayed neutrons, and β chains

1.5.1 Why neutrons are emitted

Heavy nuclei are neutron-rich: for ^{236}U,

$$\frac{N}{Z} = \frac{144}{92} \approx 1.57. \tag{1.19}$$

Stable medium-mass nuclei have smaller N/Z (closer to 1.3–1.4). Immediately after scission the fragments are still neutron-rich and highly excited; they evaporate **prompt neutrons** on a timescale $\sim 10^{-14}$–10^{-16} s. For thermal fission of ^{235}U, the average number of prompt neutrons per fission is

$$\langle \nu \rangle \approx 2.45. \tag{1.20}$$

1.5.2 Example N/Z after a typical split

For $^{140}\text{Xe} + {}^{94}\text{Sr} + 2n$:

$$\left.\frac{N}{Z}\right|_{\text{Xe}} = \frac{86}{54} \approx 1.59, \qquad \left.\frac{N}{Z}\right|_{\text{Sr}} = \frac{56}{38} \approx 1.47. \tag{1.21}$$

Both fragments remain above the β-stability line after prompt neutron emission.

1.5.3 Beta-decay chains

Neutron-rich fragments decay by successive β^- transitions,

$$n \to p + e^- + \bar{\nu}_e, \tag{1.22}$$

until they reach a stable N/Z. Examples:

$$^{140}\text{Xe} \xrightarrow{\beta^-} {}^{140}\text{Cs} \xrightarrow{\beta^-} {}^{140}\text{Ba} \xrightarrow{\beta^-} {}^{140}\text{La} \xrightarrow{\beta^-} {}^{140}\text{Ce (stable)}, \tag{1.23}$$

$$^{94}\text{Sr} \xrightarrow{\beta^-} {}^{94}\text{Y} \xrightarrow{\beta^-} {}^{94}\text{Zr (stable)}. \tag{1.24}$$

1.5.4 Delayed neutrons

Some β daughters are still neutron-unstable and emit a neutron after a β delay of seconds to minutes. Classic example:

$$^{137}\mathrm{I} \xrightarrow{\beta^-} {}^{137}\mathrm{Xe}^* \to {}^{136}\mathrm{Xe} + n \quad \text{(delayed neutron)}. \tag{1.25}$$

The average number of delayed neutrons per fission is only $\langle \nu_d \rangle \sim 0.015$–$0.02$ (about one delayed neutron per fifty fissions), but they are crucial for reactor control: they stretch the neutron lifetime in the chain reaction from 10^{-4} s (prompt only) to a human-operable timescale [6, 1, 2].

1.5.5 Prompt and delayed gamma rays

Fragments are born excited and emit **prompt γ rays** on the same fast timescale as prompt neutrons. Subsequent β daughters also radiate **delayed γ rays** as they de-excite.

1.6 Energy partition in fission

A typical total energy release of order 200 MeV per fission of ^{236}U is shared approximately as follows [6, 1]:

Channel	Energy (approx.)
Kinetic energy of fission fragments	$\sim$ 160 MeV
Kinetic energy of prompt neutrons	$\sim$ 5–15 MeV
Prompt γ rays	$\sim$ 7–10 MeV
β particles	$\sim$ 5–10 MeV
Delayed γ rays	$\sim$ 5 MeV
Antineutrinos (β decays)	$\sim$ 10–15 MeV
Delayed neutrons	$\lesssim$ 1 MeV

The bulk of the recoverable heat is the kinetic energy of the two fragments (Coulomb repulsion after scission), thermalised in the fuel. Antineutrinos escape and do not heat the reactor; about 10 MeV per fission is lost this way.

Remark

The fragment kinetic energies are nearly equal in the centre-of-mass frame for equal-mass splits; for asymmetric splits the light fragment carries more kinetic energy (same momentum, smaller mass).

1.7 Cross sections, barrier penetration, and chain reactions

1.7.1 Energy dependence of the fission cross section

Below the barrier, fission proceeds by tunneling; the cross section is exponentially small. As the compound excitation approaches and then exceeds E_f, the effective barrier thins and σ_f rises steeply, then saturates when the process is classically allowed. Resonances and intermediate structure appear when discrete compound states couple to the fission channels. For fissile nuclei, σ_f is huge at thermal energies ($\propto 1/v$ capture into a dense compound spectrum already above E_f); for ^{238}U, σ_f is small until $K_n \sim 1\,\text{MeV}$ [6, 1].

1.7.2 Chain reaction (brief)

Each fission emits $\langle \nu \rangle \sim 2.5$ neutrons. If, on average, one or more of those neutrons induces another fission, a **chain reaction** results. Define the multiplication factor k as the number of fissions in one generation divided by the number in the previous generation:

- $k < 1$: subcritical (dies out);
- $k = 1$: critical (steady power);
- $k > 1$: supercritical (power rises).

In a thermal reactor, fast fission neutrons are slowed by a moderator (water, graphite, ...) to exploit the large thermal σ_f of ^{235}U. Delayed neutrons make k controllable. Weapon designs aim for a rapid supercritical excursion with fast neutrons and highly enriched fissile material. Detailed reactor physics (four-factor formula, moderation, poisoning) lies beyond this chapter; the nuclear ingredients are the large thermal σ_f of fissile isotopes, $\langle \nu \rangle > 1$, and the existence of delayed neutrons [6, 1, 14].

Takeaway

- $n + {}^{235}$U: $Q \sim 6.6\,\text{MeV} > E_f \sim 6.2\,\text{MeV}$ (fissile). $n + {}^{238}$U: $Q \sim 4.8\,\text{MeV} < E_f$ (needs $K_n \gtrsim 1\,\text{MeV}$).
- Mass yields: asymmetric peaks near $A \sim 95, 140$; liquid drop alone insufficient.
- $Q = (E_1 - E_0)A - 2\delta a$; $\sim 200\,\text{MeV}$ total, mostly fragment KE.
- $\langle \nu \rangle_{\text{prompt}} \approx 2.45$; delayed neutrons ~ 0.02 per fission; β chains to stability.
- Chain reaction when multiplication $k \geq 1$; delayed neutrons enable control.

1.8 Further physics from the literature

Pairing and the fissile pattern. The large capture Q of $n + {}^{235}$U relative to $n + {}^{238}$U is a pairing effect: adding a neutron to odd-A ^{235}U produces even–even ^{236}U with an extra pairing energy, increasing the neutron separation energy. The same pattern explains fissile ^{239}Pu and ^{233}U [6, 1, 14].

Ternary fission and light charged particles. Occasionally a third light charged particle (α, ^{3}H, ...) is emitted at scission (ternary fission), at the 10^{-3} level. It is a probe of the neck dynamics between the two main fragments [6, 2].

Reactor antineutrinos. Each fission leads to several β^- decays and therefore several $\bar{\nu}_e$. Reactor antineutrino spectra are used for oscillation experiments and for remote monitoring of reactor power and fuel evolution [14, 2].

Evaluated nuclear data. Fission yields, $\bar{\nu}(E)$, and cross sections $\sigma_f(E)$ are tabulated in ENDF/B, JEFF, and JENDL libraries and summarised in ENSDF/NuDat for decay properties of individual fragments [20, 21].

Reading guide

Krane [6] Ch. 13 (induced fission, yields, reactors); Dunlap [1] (energetics and neutron emission); Basdevant [14] (chain reactions); Demtröder [2] (experimental systematics).

1.9 Problems with solutions

Problem 24.1 — Fissile vs fissionable

Given $Q(n + {}^{235}\mathrm{U}) = 6.55\,\mathrm{MeV}$, $E_f({}^{236}\mathrm{U}) = 6.2\,\mathrm{MeV}$, $Q(n + {}^{238}\mathrm{U}) = 4.80\,\mathrm{MeV}$, and $E_f({}^{239}\mathrm{U}) = 6.2\,\mathrm{MeV}$, determine for each case whether thermal neutrons can induce fission, and estimate the neutron kinetic energy threshold for ${}^{238}\mathrm{U}$ [6, 1].

Solution 24.1

Step 1 (${}^{235}\mathrm{U}$). Excitation $E^* \approx 6.55\,\mathrm{MeV} > 6.2\,\mathrm{MeV}$. Thermal neutrons induce fission: fissile.

Step 2 (${}^{238}\mathrm{U}$). $E^* \approx 4.80\,\mathrm{MeV} < 6.2\,\mathrm{MeV}$. Thermal fission is blocked by a residual barrier $\approx 1.4\,\mathrm{MeV}$.

Step 3 (threshold). Need $Q + K_n \gtrsim E_f$, so

$$K_n \gtrsim 6.2 - 4.8 = 1.4\,\mathrm{MeV}.$$

(Laboratory thresholds are of this order; precise values include CM corrections and barrier tunneling tails.)

Problem 24.2 — Asymmetry correction to Q

Take $A = 240$, $E_0 = 7.6\,\mathrm{MeV}$, $E_1 = 8.5\,\mathrm{MeV}$, and suppose $\delta = 0.05\,\mathrm{MeV}$ with asymmetry $a = 20$ (so fragments near $A = 100$ and 140). Compute Q_{sym} and Q_{asym} from Eq. (1.16).

Solution 24.2

Step 1.

$$Q_{\mathrm{sym}} = (8.5 - 7.6) \times 240 = 216\,\mathrm{MeV}.$$

Step 2.

$$Q_{\mathrm{asym}} = 216 - 2 \times 0.05 \times 20 = 216 - 2 = 214\,\mathrm{MeV}.$$

The asymmetric path loses only $\sim 2\,\mathrm{MeV}$ in this schematic model; shell-driven dynamics still select it [6, 14].

Problem 24.3 — Neutrons from N/Z

^{236}U has $Z = 92$, $N = 144$. Suppose it splits into fragments with $Z_1 = 54$, $Z_2 = 38$ without emitting neutrons, sharing N in proportion to A. Argue why prompt neutron emission is expected, given that stable $A \sim 90$–140 nuclei have $N/Z \sim 1.3$–1.4.

Solution 24.3

Step 1. Parent $N/Z = 144/92 \approx 1.57$.
Step 2. Conserving N/Z in each fragment leaves both with $N/Z \sim 1.57$, far above the stable valley at those Z.
Step 3. Excess neutrons are weakly bound in the hot fragments and evaporate as prompt neutrons until the neutron separation energy rises. Residual neutron richness is then removed slowly by β^- decay [1, 6].

Problem 24.4 — Energy partition

If 200 MeV is released per fission and 160 MeV appears as fragment kinetic energy, what fraction of the energy is carried by fragments? If a reactor fissions 10^{19} nuclei per second, estimate the fragment-heating power in megawatts.

Solution 24.4

Step 1. Fraction: $160/200 = 0.80$ (80%).
Step 2. Energy per second from fragments:

$$P = 10^{19} \times 160\,\text{MeV/s}.$$

Convert: $1\,\text{MeV} = 1.602 \times 10^{-13}\,\text{J}$, so

$$P = 10^{19} \times 160 \times 1.602 \times 10^{-13}\,\text{W} \approx 2.56 \times 10^{8}\,\text{W} \approx 256\,\text{MW}.$$

(Total thermal power including other channels would be $\sim 320\,\text{MW}$ before neutrino losses.)

Problem 24.5 — Multiplication factor

In a certain assembly, each fission produces on average 2.4 neutrons, and the probability that a neutron induces another fission (rather than leaking or being captured non-fissilely) is $p = 0.40$. Find the multiplication factor k and state whether the assembly is subcritical, critical, or supercritical.

Solution 24.5

$$k = \langle \nu \rangle\, p = 2.4 \times 0.40 = 0.96 < 1.$$

The assembly is subcritical: the chain dies out. Raising enrichment, reducing leakage, or improving moderation can increase p until $k = 1$ [6, 1].

Problem 24.6 — Delayed neutron fraction

Suppose $\langle \nu_{\text{prompt}} \rangle = 2.43$ and $\langle \nu_{\text{delayed}} \rangle = 0.016$. What fraction β of neutrons are delayed? Why does a small β still control reactor kinetics?

Solution 24.6

Step 1.

$$\beta = \frac{0.016}{2.43 + 0.016} \approx 0.0065 \quad (0.65\%).$$

Step 2. Prompt neutron lifetimes in a reactor core are $\sim 10^{-4}$ s or less. If k exceeds unity by more than β, the reactor is prompt supercritical and power rises uncontrollably on the prompt timescale. For $1 < k < 1 + \beta$, the excess multiplication waits on delayed neutrons (lifetimes of 0.1–50 s), so control rods can respond. Thus a sub-percent delayed fraction sets the operable window for criticality control [6, 1].

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