Chapter 1

Nuclear Fission I

"The discovery of fission, and its interpretation as a liquid-drop splitting, transformed nuclear physics from a laboratory science into a source of industrial power."

standard historical assessment [6, 14]

Nuclear fission is the splitting of a heavy nucleus into two fragments of comparable mass. Every commercial nuclear power reactor extracts energy from this process. In this chapter we define fission, recount the 1938–1939 discovery, derive the Q-value for symmetric fission from the semi-empirical mass formula (SEMF), obtain the energetic condition $Z^2/A > 17.6$, and explain why nuclei that are energetically free to fission still do not spontaneously split: the *fission barrier* of the Bohr–Wheeler liquid-drop picture. Neutron-induced fission and the compound-nucleus mechanism close the discussion and prepare Lecture 25 (this volume, Chapter **??**) [6, 1, 14, 2].

1.1 Definition and energy scale

Fission is the nuclear reaction in which a heavy parent X splits into two roughly equal fragments Y and Z,

$$X \;\to\; Y + Z \qquad (A \approx A_1 + A_2,\; Z \approx Z_1 + Z_2). \tag{1.1}$$

"Roughly equal" means the two fragments have comparable mass numbers; the extreme opposite is light-particle emission (α, proton, neutron), already treated in earlier chapters. In practice a few free neutrons are also emitted, so a more complete schematic is $X \to Y + Z + \nu n$ with $\nu \sim 2$–3. The essential physics is still the binary split of a heavy system into two medium-mass nuclei [6, 1].

Key idea

Fission: a heavy nucleus divides into two fragments of comparable mass ($A_1 \sim A_2$). Light-particle emission is *not* fission. Typical energy release for $A \sim 240$ is of order $200\,\mathrm{MeV}$ per event.

Why so much energy? The binding-energy-per-nucleon curve B/A versus A rises from light nuclei to a broad maximum near iron and nickel ($A \sim 56$–62, $B/A \sim 8.7$–$8.8\,\mathrm{MeV}$) and then *declines* slowly for heavy nuclei. For uranium-region nuclei ($A \sim 230$–240) one has $B/A \sim 7.6\,\mathrm{MeV}$; for fragments near $A \sim 100$–140 one finds $B/A \sim 8.5\,\mathrm{MeV}$. Moving nucleons from a low-B/A parent into higher-B/A daughters releases energy of order

$$Q \sim \left[(B/A)_{\text{frag}} - (B/A)_{\text{parent}}\right] A \sim (8.5 - 7.6)\,\text{MeV} \times 240 \approx 216\,\text{MeV}. \tag{1.2}$$

That is the characteristic fission energy scale. A full SEMF derivation of when $Q > 0$ occupies Section 1.4.

1.2 Historical path: neutrons, barium, and the liquid drop

Historical note

1932–1938: neutron irradiation and the search for transuranics. Chadwick's discovery of the neutron (1932) opened a new experimental tool: neutrons have no charge, so they are not repelled by the nuclear Coulomb field and can enter heavy nuclei even at thermal energies. Fermi's group in Rome systematically bombarded elements with neutrons, hoping that n capture followed by β^- decay would produce elements beyond uranium (transuranics). Many new radioactivities appeared; their chemical identification was difficult [6, 2].

1938–1939: Hahn and Strassmann. Otto Hahn and Fritz Strassmann irradiated natural uranium with neutrons and performed meticulous radiochemistry. Among the products they found an element that behaved chemically like barium ($Z = 56$), not like a heavy transuranic homologue of radium. After repeated checks they concluded that barium itself was present: a medium-mass element had been produced from uranium ($Z = 92$) [6, 14].

1939: Meitner, Frisch, and Bohr–Wheeler. Lise Meitner and Otto Frisch interpreted the result as a binary split of the uranium nucleus, by analogy with a charged liquid drop that divides when deformed. They estimated the energy release from the mass defect and named the process *fission*. Niels Bohr and John Wheeler then developed the liquid-drop theory of the fission barrier (1939), which still organises the subject [6, 1, 14].

The chemical surprise (barium from uranium) forced a change of paradigm: neutron irradiation of heavy nuclei does not only build slightly heavier species; it can also *break* the nucleus into two pieces of intermediate mass.

1.3 Binding energy and the Q-value of fission

Write the binary fission of a nucleus ${}^{A}_{Z}X_N$ as

$${}^{A}_{Z}X_N \;\rightarrow\; {}^{A_1}_{Z_1}Y_{N_1} + {}^{A_2}_{Z_2}Z_{N_2}, \tag{1.3}$$

with

$$A = A_1 + A_2, \qquad Z = Z_1 + Z_2, \qquad N = N_1 + N_2. \tag{1.4}$$

(We omit free neutrons for the mass-energy bookkeeping; they can be restored later without changing the logic.) Nuclear masses in energy units are

$$Mc^2 = Z\,m_p c^2 + N\,m_n c^2 - E_B, \tag{1.5}$$

where E_B is the total binding energy. The reaction Q-value is the decrease in rest mass,

$$Q = (M_X - M_Y - M_Z)c^2. \tag{1.6}$$

Substitute (1.5) for each nucleus. The proton and neutron mass terms cancel by charge and nucleon conservation, leaving

$$Q = E_B(Y) + E_B(Z) - E_B(X). \tag{1.7}$$

Fission releases energy if and only if the sum of the fragment binding energies exceeds the parent binding energy: the fragments are more tightly bound per nucleon (and overall) than the parent.

Numerical estimate from B/A. Take $E_B(X) = (B/A)_X\,A$ with $(B/A)_X \approx 7.6\,\text{MeV}$ and $E_B(Y) + E_B(Z) \approx 8.5\,\text{MeV} \times (A_1 + A_2) = 8.5\,\text{MeV} \times A$. Then

$$Q \approx (8.5 - 7.6)\,\text{MeV} \times A = 0.9\,\text{MeV} \times A. \tag{1.8}$$

For $A \approx 240$,

$$Q \approx 0.9 \times 240 = 216\,\text{MeV}, \tag{1.9}$$

as anticipated in (1.2). Systems tend toward lower rest-mass energy; fission of heavy nuclei is therefore energetically favoured once the fragments sit on the high-B/A side of the binding curve [6, 1, 2].

Remark

Equation (1.7) is general: any rearrangement of nucleons that increases total binding releases energy. Fusion of light nuclei (Chapters **??**–**??**) uses the same identity with the opposite motion on the B/A curve.

1.4 Symmetric fission from the SEMF: full derivation

To find the *minimum* mass number for which fission is energetically possible without external energy, use the SEMF atomic (or nuclear) mass formula, neglecting pairing for a first estimate [6, 1, 14]:

$$M(A, Z) = Zm_p + Nm_n - a_vA + a_sA^{2/3} + a_c\frac{Z^2}{A^{1/3}} + a_{\text{sym}}\frac{(A - 2Z)^2}{A}. \tag{1.10}$$

Consider *symmetric* fission into two identical fragments,

$$A_1 = A_2 = \frac{A}{2}, \qquad Z_1 = Z_2 = \frac{Z}{2}. \tag{1.11}$$

The Q-value in mass units is

$$Q = M(A, Z) - 2\,M\left(\frac{A}{2}, \frac{Z}{2}\right). \tag{1.12}$$

1.4.1 Term-by-term cancellation

Volume term.

$$-a_vA - 2\left(-a_v\frac{A}{2}\right) = -a_vA + a_vA = 0.$$

Volume energy is extensive; it cancels for any binary split that conserves A.

Asymmetry term. For the parent,

$$a_{\rm sym}\frac{(A-2Z)^2}{A}.$$

For one fragment,

$$a_{\rm sym}\frac{\big(A/2-2(Z/2)\big)^2}{A/2}=a_{\rm sym}\frac{\big((A-2Z)/2\big)^2}{A/2}=a_{\rm sym}\frac{(A-2Z)^2}{4}\cdot\frac{2}{A}=a_{\rm sym}\frac{(A-2Z)^2}{2A}.$$

Two fragments contribute

$$2\times a_{\rm sym}\frac{(A-2Z)^2}{2A}=a_{\rm sym}\frac{(A-2Z)^2}{A},$$

which exactly cancels the parent asymmetry term. Thus the symmetry energy drops out of Q for *symmetric* charge and mass partition.

Surface term.

$$Q_s=a_sA^{2/3}-2\,a_s\left(\frac{A}{2}\right)^{2/3}=a_sA^{2/3}\left[1-2\cdot\left(2^{-1}\right)^{2/3}\right]=a_sA^{2/3}(1-2^{1/3}).$$

Since $2^{1/3}\approx 1.260$,

$$Q_s=a_sA^{2/3}(1-1.260)=-0.260\,a_s\,A^{2/3}. \tag{1.13}$$

The surface term is *negative* in Q: two spheres have more total surface area than one sphere of equal volume, so surface energy opposes fission.

Coulomb term.

$$Q_c=a_c\frac{Z^2}{A^{1/3}}-2\,a_c\frac{(Z/2)^2}{(A/2)^{1/3}}=a_c\frac{Z^2}{A^{1/3}}-2\,a_c\frac{Z^2/4}{A^{1/3}\,2^{-1/3}}=a_c\frac{Z^2}{A^{1/3}}-a_c\frac{Z^2}{2A^{1/3}}\,2^{1/3}.$$

Using $2^{1/3}/2=2^{-2/3}\approx 0.630$,

$$Q_c=a_c\frac{Z^2}{A^{1/3}}\big(1-2^{-2/3}\big)=a_c\frac{Z^2}{A^{1/3}}\big(1-0.630\big)=0.370\,a_c\,\frac{Z^2}{A^{1/3}}. \tag{1.14}$$

The Coulomb term is *positive* in Q: separating the charge into two smaller nuclei lowers the electrostatic self-energy and favours fission.

1.4.2 Net Q and the condition $Q>0$

Collecting surface and Coulomb contributions,

$$\boxed{Q=-0.26\,a_s\,A^{2/3}+0.37\,a_c\,\frac{Z^2}{A^{1/3}}} \tag{1.15}$$

(pairing neglected; volume and symmetry cancelled). Spontaneous fission is energetically allowed when $Q > 0$:

$$0.37\, a_c \frac{Z^2}{A^{1/3}} > 0.26\, a_s\, A^{2/3}, \tag{1.16}$$

or equivalently

$$\frac{Z^2}{A} > \frac{0.26\, a_s}{0.37\, a_c}. \tag{1.17}$$

With standard SEMF coefficients $a_s \approx 18\,\mathrm{MeV}$ and $a_c \approx 0.72\,\mathrm{MeV}$ (order of the usual Weizsäcker values; slight variations among textbooks do not change the conclusion),

$$\frac{0.26 \times 18}{0.37 \times 0.72} \approx \frac{4.68}{0.266} \approx 17.6. \tag{1.18}$$

Hence the classic energetic criterion

$$\boxed{\frac{Z^2}{A} > 17.6 \quad \text{(symmetric fission energetically possible).}} \tag{1.19}$$

For nuclei along the valley of stability, $Z/A \sim 0.4$–0.45 at large A, so $Z^2/A \sim 0.16\,A$ to $0.20\,A$. The inequality $Z^2/A > 17.6$ is then satisfied for

$$A \gtrsim 100 \tag{1.20}$$

(roughly). All nuclei with $A \gtrsim 100$ are, on pure rest-mass grounds, candidates for spontaneous fission into two medium fragments [6, 1, 14, 2].

Key idea

Symmetric fission Q from SEMF: $Q = -0.26\, a_s A^{2/3} + 0.37\, a_c Z^2/A^{1/3}$. $Q > 0$ when $Z^2/A > 17.6$. Energetically, $A \gtrsim 100$ can fission. Volume and symmetry terms cancel; surface opposes, Coulomb drives fission.

Caution

Energetically allowed does *not* mean observed. Uranium and thorium have $A > 100$ and $Q > 0$, yet they are long-lived. The reason is the fission *barrier*: intermediate deformed shapes cost energy even when the final fragments lie lower than the parent (Section 1.5).

1.5 Why no spontaneous fission? The fission barrier

If $Q > 0$, why do ^{235}U and ^{238}U exist? The nucleus cannot jump discontinuously from a single sphere to two well-separated fragments. It must pass through a continuous sequence of deformed shapes at essentially constant volume (nuclear matter is nearly incompressible). Along that path the total energy $E(r)$ of the liquid drop first *rises*, then falls toward the asymptotic Coulomb repulsion of two fragments. The maximum of $E(r)$ relative to the ground-state energy is the **activation energy** or **fission barrier height** E_f [6, 1, 14].

Physical competition. As the drop elongates:

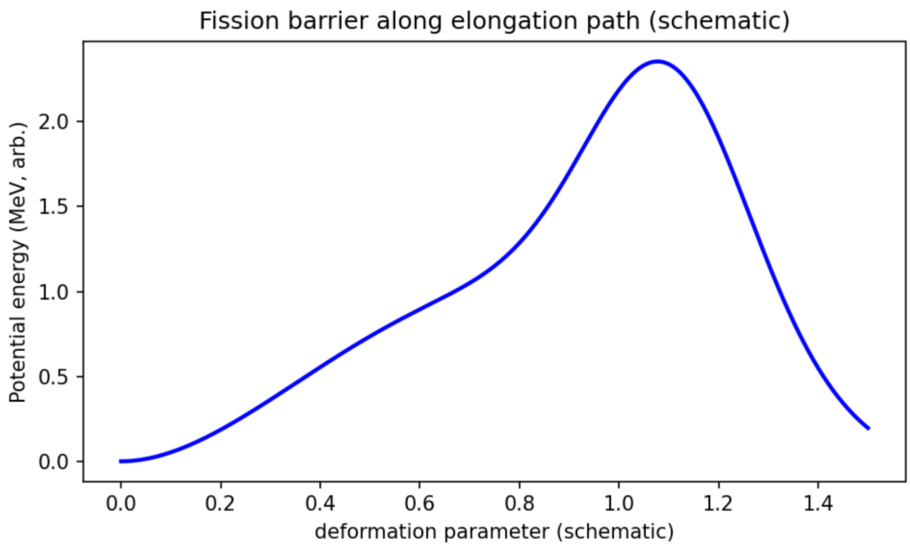

Figure 1.1: Schematic fission barrier: energy versus deformation (or fragment separation). The parent ground state sits in a well; a barrier of height E_f separates it from the scission region and the $1/r$ Coulomb repulsion of two fragments. For $A \sim 240$, $E_f \sim 5.5$–6.5 MeV. Re-extracted from the original PH5001 lecture source (Prof. Bharat Kumar, PH5001).

- **Surface energy increases** (larger surface area at fixed volume).
- **Coulomb energy decreases** (protons move farther apart on average).

Near sphericity the surface penalty dominates for moderate Z^2/A, so $E(r)$ rises. At large separation the Coulomb repulsion between two fragments dominates and $E(r) \to Z_1 Z_2 e^2/(4\pi\varepsilon_0 r)$ (taking the zero of energy at infinite separation). Between these limits a barrier peak appears (Fig. 1.1).

Energy reference. It is convenient to set $E = 0$ when the two fragments are infinitely far apart (no nuclear interaction, vanishing Coulomb energy). For $A \sim 240$ the ground-state rest energy of the intact nucleus then lies about $Q \sim 200$ MeV *above* that zero: fission lowers the total energy by Q, but only after the system has climbed the intermediate barrier of height $E_f \sim 6$ MeV.

Barrier height for heavy nuclei. Bohr and Wheeler estimated E_f from the liquid-drop energy as a function of deformation. For uranium-region nuclei one finds typically

$$E_f \sim 5.5\text{–}6.5\,\text{MeV}. \tag{1.21}$$

Spontaneous fission then requires either (i) thermal or quantum tunneling through a barrier of several MeV, which is extremely slow for $A \sim 240$, or (ii) supply of excitation energy comparable to E_f by a reaction (neutron capture, photon absorption, ...). That is why uranium is metastable on geological timescales despite $Q > 0$ [6, 1].

1.6 Deformation energy: ellipsoid calculation

A quantitative sketch of the barrier near sphericity follows Bohr and Wheeler. Start with a spherical nucleus of radius R and deform it into a prolate ellipsoid of semi-axes a (long) and $b = b$ (equal short axes), at *constant volume*:

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi ab^2 \quad \Rightarrow \quad R^3 = ab^2. \tag{1.22}$$

Parametrise a small elongation by

$$a = R(1+\varepsilon), \qquad b = \frac{R}{\sqrt{1+\varepsilon}} \tag{1.23}$$

(so that $ab^2 = R^3$ to first order; for small ε one often uses the equivalent second-order expansion $b \approx R(1-\varepsilon/2)$).

To second order in the deformation parameter ε, the surface area of the ellipsoid is

$$S \approx S_0\left(1 + \frac{2}{5}\varepsilon^2\right), \tag{1.24}$$

where $S_0 = 4\pi R^2$ is the sphere area. The surface energy therefore becomes

$$E_{\text{surf}} = a_s A^{2/3}\left(1 + \frac{2}{5}\varepsilon^2\right). \tag{1.25}$$

The Coulomb self-energy of a uniformly charged ellipsoid, to the same order, is

$$E_{\text{Coul}} = a_c \frac{Z^2}{A^{1/3}}\left(1 - \frac{1}{5}\varepsilon^2\right). \tag{1.26}$$

(Surface grows; Coulomb falls as charge spreads along the long axis.)

The change in energy relative to the sphere is

$$\Delta E(\varepsilon) = E(\varepsilon) - E(0) = a_s A^{2/3} \cdot \frac{2}{5}\varepsilon^2 - a_c \frac{Z^2}{A^{1/3}} \cdot \frac{1}{5}\varepsilon^2 = \frac{\varepsilon^2}{5}\left[2a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}\right]. \tag{1.27}$$

For the sphere to be *unstable* against infinitesimal deformation ($\Delta E < 0$ for any small $\varepsilon \neq 0$), one needs

$$2a_s A^{2/3} < a_c \frac{Z^2}{A^{1/3}}, \tag{1.28}$$

or

$$\frac{Z^2}{A} > \frac{2a_s}{a_c}. \tag{1.29}$$

With $a_s \approx 16.4\,\text{MeV}$ and $a_c \approx 0.72\,\text{MeV}$ (coefficients of the type used in the lecture for this estimate),

$$\frac{2a_s}{a_c} \approx \frac{32.8}{0.72} \approx 46\text{–}47. \tag{1.30}$$

Thus

$$\frac{Z^2}{A} \gtrsim 47 \tag{1.31}$$

marks the loss of a barrier against small deformations: the nucleus is unstable at sphericity and fissions spontaneously with no activation energy. Taking $Z \approx 0.4A$ gives $Z^2/A \approx 0.16A$, so

$$A \gtrsim 300 \tag{1.32}$$

for barrierless spontaneous fission. No such nuclei exist as long-lived species; they would fission on nuclear timescales. Between the energetic threshold $Z^2/A \sim 18$ ($A \sim 100$) and the absolute instability $Z^2/A \sim 47$ ($A \sim 300$), nuclei are metastable: $Q > 0$ but $E_f > 0$ [6, 1, 14].

Remark

The ratio $x = (Z^2/A)/(Z^2/A)_{\text{crit}}$ is the classical *fissility* parameter. For uranium, $x \sim 0.7$–0.8: a finite barrier remains, but it is only a few MeV high.

1.7 Photofission and neutron-induced fission

1.7.1 Photofission

A γ ray deposits energy without changing A or Z:

$$\gamma + {}^{A}X \;\to\; \text{fission products.} \tag{1.33}$$

If the photon energy exceeds E_f, the nucleus can surmount the barrier and fission. Photofission is a clean experimental probe of the activation energy: no nucleon is added, so the barrier refers to the original nucleus. The fission cross section rises steeply with E_γ as the effective barrier width and height for tunneling shrink, then saturates once the classical threshold is passed [6, 1].

1.7.2 Neutron-induced fission and the compound nucleus

A neutron can induce fission by forming a compound nucleus that is already excited:

$$n + {}^{A}X \;\to\; {}^{A+1}X^{*} \;\to\; \text{fission products.} \tag{1.34}$$

The excitation energy of the compound system is essentially the capture Q-value plus the kinetic energy of the neutron (centre-of-mass corrections are small for heavy targets). If that excitation exceeds E_f, fission is likely; if not, the system must tunnel, and the fission cross section remains tiny. Competing channels include elastic scattering and radiative capture (n, γ).

Whether thermal neutrons suffice depends on the capture Q-value relative to the barrier of the compound nucleus. That comparison for ^{235}U versus ^{238}U is the central theme of the next chapter [6, 1, 2].

Takeaway

- Fission: heavy $X \to Y + Z$ with $A_Y \sim A_Z$; $Q \sim 200\,\text{MeV}$ for $A \sim 240$ from the rise in B/A.
- History: neutron discovery (1932); Hahn–Strassmann barium (1939); Meitner–Frisch

interpretation; Bohr–Wheeler barrier theory.

- SEMF symmetric $Q = -0.26 a_s A^{2/3} + 0.37 a_c Z^2/A^{1/3}$; $Q > 0$ for $Z^2/A > 17.6$ ($A \gtrsim 100$).
- Barrier from surface vs Coulomb along a deformation path; $E_f \sim 6\,\text{MeV}$ for $A \sim 240$; no barrier for $Z^2/A \gtrsim 47$ ($A \gtrsim 300$).
- Induced fission: photofission measures E_f; neutron capture forms B^* with excitation $\approx Q_{\text{capture}} + K_n$.

1.8 Further physics from the literature

Asymmetric fission and shell corrections. The pure liquid drop predicts a preference for *symmetric* mass splits near the barrier, yet thermal-neutron fission of ^{235}U is strongly asymmetric (peaks near $A \sim 95$ and $A \sim 140$). Strutinsky shell corrections to the liquid-drop energy create secondary minima and drive the mass-asymmetric valleys observed experimentally. Chapter **??** returns to yield curves; the microscopic theory is developed in advanced texts [6, 14, 2].

Spontaneous fission half-lives. Even with $Q > 0$, spontaneous fission half-lives range from milliseconds (very heavy Z) to 10^{15} years and beyond. Systematics correlate roughly with Z^2/A: larger fissility, lower barrier, shorter half-life. Pairing and shell structure produce large odd–even staggering [6, 1, 20].

Double-humped barriers. For actinides the potential energy surface often shows two barriers with a secondary minimum (shape isomer). Delayed fission and intermediate structure in fission cross sections are experimental signatures. The simple single-barrier sketch of Fig. 1.1 is the first approximation [6, 14].

Historical reading. Meitner and Frisch's 1939 note and Bohr and Wheeler's physical review article remain readable introductions. Modern textbook accounts that follow the same SEMF algebra as this chapter include Krane Ch. 13, Dunlap's fission chapter, and Basdevant *et al.* [6, 1, 14].

Reading guide

Krane [6] Ch. 13 (energetics and barrier); Dunlap [1] (liquid drop and Q); Basdevant [14] (fissility and deformation); Demtröder [2] (historical and experimental overview). Evaluated fission data: ENSDF and NuDat 3 [20, 21].

1.9 Problems with solutions

Problem 23.1 — Q from B/A for $A = 238$

Estimate the fission Q-value for a nucleus with $A = 238$ if $(B/A)_{\text{parent}} = 7.6\,\text{MeV}$ and $(B/A)_{\text{fragments}} = 8.5\,\text{MeV}$. Express the result in MeV and comment on its magnitude relative to chemical combustion energies [6, 1].

Solution 23.1

Step 1. From Eq. (1.7),

$$Q = E_B(Y) + E_B(Z) - E_B(X) \approx \left[(B/A)_{\text{frag}} - (B/A)_{\text{parent}}\right] A.$$

Step 2. Substitute the given values:

$$Q \approx (8.5 - 7.6)\,\text{MeV} \times 238 = 0.9 \times 238 = 214\,\text{MeV}.$$

Step 3. Chemical energies are a few eV per reaction. Fission releases $\sim 10^8$ times more energy per event. That is why a few kilograms of fissile material can power a reactor or a weapon [6].

Problem 23.2 — Symmetric SEMF Q algebra

Starting from $M(A, Z)$ in Eq. (1.10) (pairing omitted), derive $Q = -0.26\,a_s A^{2/3} + 0.37\,a_c Z^2/A^{1/3}$ for symmetric fission into two equal fragments. Show that volume and symmetry terms cancel [14, 1].

Solution 23.2

Step 1. $Q = M(A, Z) - 2M(A/2, Z/2)$.
Step 2 (volume). $-a_v A - 2(-a_v A/2) = 0$.
Step 3 (symmetry). Parent: $a_{\text{sym}}(A-2Z)^2/A$. One fragment: $a_{\text{sym}}(A/2-Z)^2/(A/2) = a_{\text{sym}}(A-2Z)^2/(2A)$. Two fragments: $a_{\text{sym}}(A-2Z)^2/A$. Difference vanishes.
Step 4 (surface).

$$a_s A^{2/3} - 2a_s(A/2)^{2/3} = a_s A^{2/3}(1 - 2^{1/3}) = -0.260\,a_s A^{2/3}.$$

Step 5 (Coulomb).

$$a_c \frac{Z^2}{A^{1/3}} - 2a_c \frac{(Z/2)^2}{(A/2)^{1/3}} = a_c \frac{Z^2}{A^{1/3}}(1 - 2^{-2/3}) = 0.370\,a_c \frac{Z^2}{A^{1/3}}.$$

Step 6. Add surface and Coulomb contributions to obtain Eq. (1.15).

Problem 23.3 — Critical Z^2/A

Using $a_s = 18.0\,\text{MeV}$ and $a_c = 0.72\,\text{MeV}$, evaluate the threshold Z^2/A for $Q > 0$ in symmetric fission. For ^{238}U ($Z = 92$, $A = 238$), is $Q > 0$?

Solution 23.3

Step 1. From Eq. (1.17),

$$\frac{Z^2}{A} > \frac{0.26\,a_s}{0.37\,a_c} = \frac{0.26 \times 18.0}{0.37 \times 0.72} = \frac{4.68}{0.2664} \approx 17.6.$$

Step 2. For ^{238}U,

$$\frac{Z^2}{A} = \frac{92^2}{238} = \frac{8464}{238} \approx 35.6 > 17.6.$$

Yes, $Q > 0$. Uranium is energetically free to fission but is held by the barrier E_f [6, 1].

Problem 23.4 — Barrierless fissility

From the small-deformation result $\Delta E \propto [2a_s A^{2/3} - a_c Z^2/A^{1/3}]$, show that the sphere is unstable when $Z^2/A > 2a_s/a_c$. With $a_s = 16.4\,\text{MeV}$ and $a_c = 0.72\,\text{MeV}$, estimate the critical A if $Z = 0.4A$.

Solution 23.4

Step 1. Instability requires $\Delta E < 0$:

$$2a_s A^{2/3} < a_c Z^2/A^{1/3} \quad \Rightarrow \quad \frac{Z^2}{A} > \frac{2a_s}{a_c}.$$

Step 2.

$$\frac{2a_s}{a_c} = \frac{32.8}{0.72} \approx 45.6.$$

Step 3. With $Z = 0.4A$, $Z^2/A = 0.16A$. Set $0.16A = 45.6 \Rightarrow A \approx 285$. Order-of-magnitude result: nuclei with $A \gtrsim 300$ have essentially no fission barrier and cannot exist as long-lived species [14, 6].

Problem 23.5 — Activation and thermal energy

The fission barrier of a heavy nucleus is $E_f = 6.0\,\text{MeV}$. Compare this energy with $k_B T$ at room temperature ($T = 300\,\text{K}$) and estimate the Boltzmann factor $e^{-E_f/k_B T}$. What does this imply for thermal spontaneous fission?

Solution 23.5

Step 1.

$$k_B T = (8.617 \times 10^{-5}\,\text{eV/K}) \times 300\,\text{K} \approx 0.026\,\text{eV} = 2.6 \times 10^{-8}\,\text{MeV}.$$

Step 2.

$$\frac{E_f}{k_B T} = \frac{6.0}{2.6 \times 10^{-8}} \approx 2.3 \times 10^8.$$

The Boltzmann factor $e^{-E_f/k_B T}$ is utterly negligible. Thermal activation over the barrier does not occur at laboratory temperatures; spontaneous fission proceeds only by quantum tunneling (extremely slow for $A \sim 240$) or by nuclear reactions that supply MeV-scale excitation [6, 1].

Problem 23.6 — Surface vs Coulomb for ε

Using Eqs. (1.25)–(1.27), compute ΔE (in MeV) for ^{238}U at $\varepsilon = 0.1$. Take $a_s = 18\,\text{MeV}$, $a_c = 0.72\,\text{MeV}$, $Z = 92$, $A = 238$. Is the sphere stable against this deformation?

Solution 23.6

Step 1.

$$A^{2/3} = 238^{2/3} \approx 38.3, \qquad A^{1/3} \approx 6.20.$$

Step 2.

$$\Delta E = \frac{(0.1)^2}{5}\left[2 \times 18 \times 38.3 - 0.72 \times \frac{8464}{6.20}\right] = 0.002[1379 - 983] = 0.002 \times 396 \approx 0.79\,\text{MeV}.$$

Step 3. $\Delta E > 0$: the sphere is stable against this small deformation; a finite barrier remains. (The full barrier height requires a global calculation beyond the quadratic expansion [6, 14].)

Bibliography

[1] R. A. Dunlap, *An Introduction to the Physics of Nuclei and Particles* (2nd ed.), IOP Publishing, Bristol (2023), DOI: 10.1088/978-0-7503-6094-4, Ch. 3 (Nuclear composition and size).

[2] W. Demtröder, *Nuclear and Particle Physics*, Undergraduate Lecture Notes in Physics, Springer, Cham (2023).

[3] S. Petrera, *Problems and Solutions in Nuclear and Particle Physics*, UNITEXT for Physics, Springer, Cham (2019).

[4] F. Terranova, *A Modern Primer in Particle and Nuclear Physics*, Oxford University Press (2021).

[5] S. D'Auria, *Introduction to Nuclear and Particle Physics*, Undergraduate Lecture Notes in Physics, Springer, Cham (2018).

[6] K. S. Krane, *Introductory Nuclear Physics*, Wiley, New York (1988).

[7] R. Hofstadter, "Electron scattering and nuclear structure," Rev. Mod. Phys. **28**, 214 (1956).

[8] H. F. Ehrenberg, R. Hofstadter, U. Meyer-Berkhout, D. G. Ravenhall, and S. E. Sobottka, "High-Energy Electron Scattering and the Charge Distribution of Carbon-12 and Oxygen-16," Phys. Rev. **113**, 666 (1959).

[9] H. Yukawa, "On the interaction of elementary particles. I," Proc. Phys.-Math. Soc. Japan **17**, 48 (1935).

[10] R. D. Woods and D. S. Saxon, "Diffuse surface optical model for nucleon-nuclei scattering," Phys. Rev. **95**, 577 (1954).

[11] H. G. J. Moseley, "The high-frequency spectra of the elements," Phil. Mag. **26**, 1024 (1913); **27**, 703 (1914).

[12] C. F. von Weizsäcker, "Zur Theorie der Kernmassen," Z. Phys. **96**, 431 (1935).

[13] P. Cappellaro, *Introduction to Applied Nuclear Physics* (MIT OpenCourseWare / LibreTexts), CC BY-NC-SA 4.0.

[14] J.-L. Basdevant, J. Rich, and M. Spiro, *Fundamentals in Nuclear Physics: From Nuclear Structure to Cosmology*, Springer, New York (2005).

[15] D. Tong, *Particle Physics*, Lecture notes (University of Cambridge / CERN school notes).

[16] M. G. Mayer, "On closed shells in nuclei. II," Phys. Rev. **75**, 1969 (1949).

[17] O. Haxel, J. H. D. Jensen, and H. E. Suess, "On the "magic numbers" in nuclear structure," Phys. Rev. **75**, 1766 (1949).

[18] T. Schmidt, "Über die magnetischen Momente der Atomkerne," Z. Phys. **106**, 358 (1937).

[19] L. W. Nordheim, "On spins, moments, and shells in nuclei," Phys. Rev. **78**, 294 (1950).

[20] Evaluated Nuclear Structure Data File (ENSDF), National Nuclear Data Center, Brookhaven National Laboratory, `https://www.nndc.bnl.gov/ensdf/`.

[21] NuDat 3, National Nuclear Data Center, Brookhaven National Laboratory, `https://www.nndc.bnl.gov/nudat3/`.

[22] N. J. Stone, "Table of nuclear magnetic dipole and electric quadrupole moments," At. Data Nucl. Data Tables **90**, 75 (2005); updated compilations at NNDC.

[23] W. N. Cottingham and D. A. Greenwood, *An Introduction to Nuclear Physics* (2nd ed.), Cambridge University Press, Cambridge (2001).

[24] H. Geiger and J. M. Nuttall, "The ranges of the α particles from the radioactive substances," Phil. Mag. **22**, 613 (1911).

[25] G. Gamow, "Zur Quantentheorie des Atomkernes," Z. Phys. **51**, 204 (1928).

[26] R. W. Gurney and E. U. Condon, "Wave mechanics and radioactive disintegration," Nature **122**, 439 (1928); Phys. Rev. **33**, 127 (1929).

[27] M. Wang, W. J. Huang, F. G. Kondev, G. Audi, and S. Naimi, "The AME 2020 atomic mass evaluation (II). Tables, graphs and references," Chin. Phys. C **45**, 030003 (2021).

[28] Laboratoire National Henri Becquerel (LNHB), Recommended decay data, `http://www.lnhb.fr/nuclear-data/nuclear-data-table/`.

[29] L. M. Langer, R. D. Moffat, and H. C. Price, Jr., "Low energy beta-ray spectra: P^{32} and Cu^{64}," Phys. Rev. **76**, 1725 (1949).

[30] L. M. Langer and H. C. Price, Jr., "Shapes of the beta-spectra of the forbidden transitions of Sb^{124}, Rb^{86}, Sr^{89}, Y^{91}, and Cs^{137}," Phys. Rev. **75**, 1769 (1949).

[31] D. C. Camp and L. M. Langer, "The shape of the ^{66}Ga positron spectrum and the contribution from $0^+ \to 0^+$ transitions," Phys. Rev. **129**, 1782 (1963).