Chapter 1

Gamma Decay

> *"Gamma radiation is the electromagnetic de-excitation of the nucleus: same species, lower energy, multipole selection rules, and competition from internal conversion."*
>
> nuclear-structure perspective [6, 1]

Alpha and beta decay change the nuclear identity (Z and/or A). **Gamma decay** does not: the same nucleus drops from an excited state to a lower level by emitting a photon. Excited states are populated most often by preceding β or α decay (or by nuclear reactions). The transition energy is typically tens of keV to a few MeV. This chapter develops multipole classification (EL, ML), angular-momentum and parity selection rules, internal conversion, and the competition that assigns preferred multipoles, following Krane, Dunlap, Basdevant, and Demtröder [6, 1, 14, 2].

1.1 What gamma decay is

A nucleus X in an excited state X^* of excitation energy E^* relative to a lower state (often the ground state) may emit a photon:

$$X^* \longrightarrow X + \gamma. \tag{1.1}$$

Neither Z nor A changes. The available energy is the level difference (recoil corrections appear in Section 1.9). The process is the nuclear analogue of an atomic optical transition: discrete levels, photon energy set by ΔE, and selection rules from angular momentum and parity.

How the nucleus becomes excited. Typical routes are

- β^-, β^+, or electron capture to an excited daughter;
- α decay to an excited daughter;
- inelastic scattering or capture reactions.

The subsequent γ cascade maps the level scheme of the daughter [6, 1].

1.2 Standard examples: ^{137}Cs, ^{60}Co, ^{22}Na

1.2.1 ^{137}Cs $\to$ ^{137}Ba

Cesium-137 ($Z = 55$) β^--decays to barium-137 ($Z = 56$). About 95% of the decays feed the 662 keV isomeric state of ^{137}Ba; about 5% go to the ground state. The isomer has half-life $t_{1/2} = 2.55$ min and de-excites by a 661.7 keV γ ray:

$$^{137}\mathrm{Cs} \xrightarrow{\beta^-} {}^{137}\mathrm{Ba}^* \xrightarrow{\gamma\,(662\,\mathrm{keV})} {}^{137}\mathrm{Ba}. \tag{1.2}$$

This line is a standard detector-calibration source [6, 20, 21].

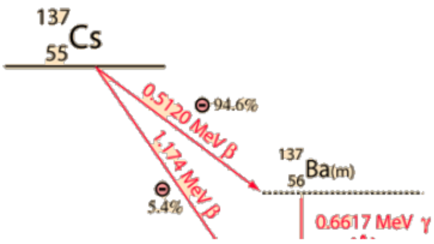

Figure 1.1: Schematic decay of ^{137}Cs to ^{137}Ba: dominant β branch to the 662 keV level, followed by γ emission (lecture scheme).

1.2.2 ^{60}Co $\to$ ^{60}Ni

Cobalt-60 β^--decays ($t_{1/2} = 5.27$ y) almost always to the 2.505 MeV, 4^+ state of ^{60}Ni. The cascade is

$$2.505\,\mathrm{MeV}\,(4^+) \xrightarrow{1.173\,\mathrm{MeV}\,\gamma} 1.332\,\mathrm{MeV}\,(2^+) \xrightarrow{1.332\,\mathrm{MeV}\,\gamma} 0\,(0^+). \tag{1.3}$$

The two strong lines at 1.173 and 1.332 MeV make ^{60}Co another standard calibration source [6, 1].

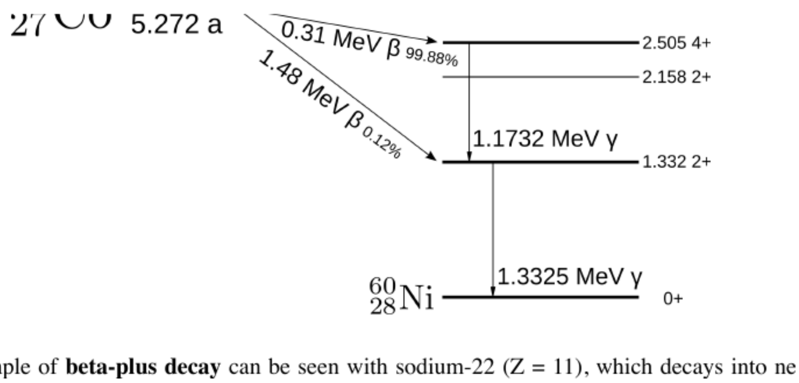

Figure 1.2: Partial level scheme of ^{60}Ni populated by ^{60}Co β decay: cascade γ rays at 1.173 and 1.332 MeV (lecture scheme).

1.2.3 ^{22}Na and β^+

Sodium-22 decays mainly by β^+ ($\sim 90\%$) and electron capture ($\sim 10\%$) to the 1.274 MeV level of ^{22}Ne, which then emits a 1.27 MeV photon. Only a tiny branch goes directly to the ground state. The positron annihilates, producing two 511 keV photons in addition to the nuclear γ ray [6, 20].

Gamma versus X-ray. Both are photons. Nuclear γ rays typically span ~ 10 keV to several MeV; atomic X-rays from K-shell transitions are usually $\lesssim 100$ keV. The ranges overlap, but the origin differs: nuclear levels versus electronic shells [1, 2].

1.3 Multipole radiation: EL and ML

Electromagnetic radiation is generated by time-varying charge and current distributions. Expanding the fields outside a localised source yields **multipoles**: electric monopole, dipole, quadrupole, ... and magnetic dipole, quadrupole, ...

No electric monopole radiation. A spherically symmetric charge distribution that oscillates radially still looks, outside the charge, like a point charge at the origin. Maxwell's equations then give no radiating $1/r$ wave. Hence there is **no electric monopole ($E0$) photon radiation**. Monopole transitions still exist as nuclear processes, but they de-excite by internal conversion or pair creation, not by a single γ ray [6, 14].

No magnetic monopoles. Classical electrodynamics has $\nabla \cdot \mathbf{B} = 0$. There is no magnetic monopole moment, so magnetic radiation starts at the **magnetic dipole** $M1$.

Notation.

- EL: electric 2^L-pole radiation ($E1$ dipole, $E2$ quadrupole, $E3$ octupole, ...);
- ML: magnetic 2^L-pole ($M1$, $M2$, ...).

The integer $L = 1, 2, 3, \ldots$ is the multipole order and equals the angular momentum (in units of $\hbar$) carried by the photon field for that multipole [6, 1].

1.4 Angular momentum selection rules

Let the initial nuclear spin be J_i and the final spin J_f. A photon of multipole order L carries angular momentum $L\hbar$. Vector addition requires the triangle inequality

$$|J_i - J_f| \ \leq \ L \ \leq \ J_i + J_f. \tag{1.4}$$

Because a real photon has helicity ± 1, $L = 0$ is forbidden for single photon emission. Allowed multipole orders therefore begin at

$$L_{\min} = \max(1,\, |J_i - J_f|) \tag{1.5}$$

when $J_i + J_f \geq 1$. Special case:

$$J_i = J_f = 0 \quad \Rightarrow \quad \text{no single-photon transition (no } L \text{ satisfies (1.4) with } L \geq 1). \tag{1.6}$$

Thus $0^+ \to 0^+$ (and any $0 \to 0$) γ emission is strictly forbidden by angular momentum.

Physical meaning. Equation (1.4) is conservation of angular momentum for the system nucleus+photon. Large $|J_i - J_f|$ forces large L, which strongly suppresses the radiative width [6, 14, 2].

1.5 Parity selection rules

Nuclear states also have parity $\pi = \pm 1$. The multipole field has definite parity under spatial inversion:

$$\text{electric } EL: \qquad \pi_f = \pi_i\,(-1)^L, \tag{1.7}$$

$$\text{magnetic } ML: \qquad \pi_f = \pi_i\,(-1)^{L+1}. \tag{1.8}$$

Equivalently, parity *changes* for $E1, M2, E3, M4, \ldots$ and is *unchanged* for $M1, E2, M3, E4, \ldots$.

Multipole	L	Parity change?	Example
$E1$	1	yes ($\pi_f = -\pi_i$)	electric dipole
$M1$	1	no ($\pi_f = \pi_i$)	magnetic dipole
$E2$	2	no	electric quadrupole
$M2$	2	yes	magnetic quadrupole
$E3$	3	yes	electric octupole
$M3$	3	no	magnetic octupole

Remark

To assign multipolarity: (i) list all L allowed by (1.4); (ii) for each L, keep EL or ML according to whether parity changes; (iii) the lowest allowed L dominates the rate [6, 1].

1.6 Preferred multipole and rough rate hierarchy

When several multipoles are allowed, lower multipole order wins. A useful order-of-magnitude rule of thumb (Weisskopf single-particle estimates, rough scale only) is

$$\frac{\lambda(L+1)}{\lambda(L)} \sim 10^{-5} \qquad \text{(same character } E \text{ or } M), \tag{1.9}$$

and, at fixed L,

$$\frac{\lambda(EL)}{\lambda(ML)} \sim 10^2. \tag{1.10}$$

Thus $E1$ (when allowed) is extremely fast; pure $M4$ or $E5$ transitions can be slow enough that the state is an **isomer** with lifetime minutes or longer [6, 1, 14].

1.7 Worked multipole assignments

1.7.1 ^{72}Ge: $0^+ \to 0^+$ and $2^+ \to 0^+$

Germanium-72 has a 0^+ ground state, a 0^+ first excited state at 691 keV ($t_{1/2} = 0.42\,\mu$s), and a 2^+ state at 834 keV ($t_{1/2} = 3.1$ ps).

$0^+ \to 0^+$ at 691 keV. Here $J_i = J_f = 0$, so (1.4) forces $L = 0$. No photon multipole exists. Gamma emission is forbidden. The state still decays by **internal conversion** (Section 1.8): the nuclear transition energy is transferred to an atomic electron [6, 1].

$2^+ \to 0^+$ at 834 keV.

$$|2 - 0| \le L \le 2 + 0 \quad \Rightarrow \quad L = 2. \tag{1.11}$$

Both states have positive parity, so $\pi_f = \pi_i$. For $L = 2$, electric radiation satisfies $\pi_f = \pi_i(-1)^2 = \pi_i$. The transition is pure $E2$. The short half-life is consistent with a low multipole order [6].

1.7.2 ^{117}In: large ΔJ

An excited state of ^{117}In at 315 keV has $J^\pi = \frac{1}{2}^-$; the ground state is $\frac{9}{2}^+$. Then

$$\left|\tfrac{1}{2} - \tfrac{9}{2}\right| \le L \le \tfrac{1}{2} + \tfrac{9}{2} \quad \Rightarrow \quad L = 4 \text{ or } 5. \tag{1.12}$$

Parity *changes* ($- \to +$).

- For $L = 4$: magnetic parity rule $\pi_f = \pi_i(-1)^5 = -\pi_i$ matches a change. Allowed multipole: $M4$.
- For $L = 5$: electric rule $\pi_f = \pi_i(-1)^5 = -\pi_i$ also matches. Allowed multipole: $E5$.

By (1.9)–(1.10), $M4$ dominates over $E5$. The high multipole order lengthens the lifetime and can produce an isomeric state [6, 1].

Assign multipolarity for $3^- \to 0^+$

Possible L: $|3 - 0| \le L \le 3$, so $L = 3$. Parity changes. Electric $E3$ has $\pi_f = \pi_i(-1)^3 = -\pi_i$. Magnetic $M3$ would not change parity. Result: pure $E3$.

1.8 Internal conversion

1.8.1 Mechanism

In **internal conversion** (IC) the nucleus de-excites by ejecting an atomic electron (usually from the K or L shell) rather than by emitting a photon. The electron wave function overlaps the nuclear volume, so the electromagnetic multipole field can couple directly to the bound electron. The kinetic energy of the conversion electron is

$$K_e = E_{\text{tr}} - B_e, \tag{1.13}$$

where $E_{\rm tr}$ is the nuclear transition energy and B_e is the atomic binding energy of the shell. Characteristic monoenergetic electron lines result [6, 1, 2].

1.8.2 Conversion coefficient

Define partial decay constants λ_γ and $\lambda_{\rm IC}$ for photon emission and internal conversion. The **internal conversion coefficient** is

$$\alpha \equiv \frac{\lambda_{\rm IC}}{\lambda_\gamma} = \frac{N_e}{N_\gamma}, \tag{1.14}$$

with N_e and N_γ the numbers of conversion electrons and photons in the same transition. One also defines shell coefficients α_K, α_L, ... with $\alpha = \alpha_K + \alpha_L + \cdots$. The total electromagnetic width is

$$\lambda_{\rm em} = \lambda_\gamma(1+\alpha). \tag{1.15}$$

Conversion is enhanced for low $E_{\rm tr}$, high Z, and high multipole order. Even when γ emission is allowed, IC competes [6, 1].

1.8.3 $0^+ \to 0^+$: conversion only

For $0^+ \to 0^+$, $\lambda_\gamma = 0$ for single-photon emission, so the only electromagnetic de-excitation channel (below pair threshold) is internal conversion (electric monopole $E0$ operator acting on the electron). Pair production e^+e^- opens if $E_{\rm tr} > 2m_ec^2$. The 691 keV 0^+ state of ^{72}Ge is the textbook example of pure conversion decay [6, 14].

Caution

Internal conversion is *not* the photoelectric effect of a previously emitted γ ray. No photon is created as an intermediate free particle; the energy is transferred in a single nuclear–atomic electromagnetic process [6].

1.9 Nuclear recoil and resonant absorption (outline)

If the free nucleus is initially at rest and emits a photon of energy E_γ, momentum conservation requires nuclear recoil momentum $p = E_\gamma/c$. Non-relativistically,

$$E_R = \frac{p^2}{2M} = \frac{E_\gamma^2}{2Mc^2}, \tag{1.16}$$

so energy conservation gives

$$E_\gamma = E_0 - E_R \approx E_0 - \frac{E_0^2}{2Mc^2}, \tag{1.17}$$

where E_0 is the level difference. For absorption, the photon must supply $E_0 + E_R$. Free-nucleus emission and absorption lines are therefore separated by $\sim 2E_R$ (often eV scale), which can block resonance unless Doppler or natural widths overlap.

If the nucleus is bound in a crystal and E_R is smaller than a phonon quantum, a finite fraction of emissions and absorptions is **recoilless**. That is the **Mössbauer effect**, used for ultra-high-resolution nuclear spectroscopy [6, 14, 2].

Key idea

Students should remember:

- γ decay: same nucleus; $X^* \to X + \gamma$.
- Multipoles EL, ML; no $E0$ photon; no magnetic monopole.
- $|J_i - J_f| \leq L \leq J_i + J_f$, $L \geq 1$; $0 \to 0$ γ forbidden.
- Parity: EL has $\pi_f = \pi_i(-1)^L$; ML has $\pi_f = \pi_i(-1)^{L+1}$.
- Lowest allowed multipole dominates; high $L \Rightarrow$ isomers.
- Conversion coefficient $\alpha = \lambda_{\mathrm{IC}}/\lambda_\gamma$; $0^+ \to 0^+$ proceeds only by IC (or pairs).

1.10 Further physics from the literature

Weisskopf single-particle estimates. Textbook multipole rate formulae (Weisskopf units) convert measured partial half-lives into transition strengths $B(EL)$ or $B(ML)$ in single-particle units. Collective $E2$ transitions in deformed nuclei can be tens to hundreds of Weisskopf units; single-particle $M1$ strengths are often quenched [6, 1, 14].

Mixed multipoles and angular correlations. When two multipoles are allowed (for example $M1 + E2$), the mixing ratio δ is measured from γ–γ angular correlations or from conversion-coefficient ratios α_K/α_L. These data fix spins and parities of levels that are not uniquely determined by selection rules alone [6, 2].

Internal conversion tables. Modern $\alpha_K(Z, E, L)$ values are computed from relativistic atomic wave functions (Band, Rose, Hager–Seltzer, BrIcc). Comparing experimental α with theory assigns multipolarity when angular-correlation data are unavailable [6, 20].

Mössbauer spectroscopy. Recoilless emission and absorption in solids resolve hyperfine interactions (isomer shift, quadrupole splitting, magnetic Zeeman splitting) at 10^{-8} eV scales relative to $E_\gamma \sim 10\,\mathrm{keV}$. The classic ^{57}Fe 14.4 keV line is the workhorse of materials science [6, 14, 2].

Reading guide

Krane [6] Ch. 10 (multipoles, conversion, Weisskopf estimates); Dunlap [1] (decay schemes and calibration sources); Basdevant [14] (electromagnetic multipoles); Demtröder [2] (Mössbauer effect and applications). Evaluated schemes: ENSDF / NuDat 3 [20, 21].

1.11 Problems with solutions

Problem 20.1: Multipole for $2^+ \to 0^+$

A nuclear transition proceeds from $J_i^\pi = 2^+$ to $J_f^\pi = 0^+$. List all multipoles allowed by angular momentum and parity, and identify the dominant one [6].

Solution 20.1

Step 1: Angular momentum.

$$|2-0| \le L \le 2+0 \quad \Rightarrow \quad L = 2.$$

Only quadrupole radiation is allowed.

Step 2: Parity. Both states have $\pi = +1$, so parity does not change. For $L = 2$,

$$EL: \ \pi_f = \pi_i(-1)^2 = \pi_i \quad \text{(allowed)}, \qquad ML: \ \pi_f = \pi_i(-1)^3 = -\pi_i \quad \text{(forbidden)}.$$

Step 3: Dominant multipole. The only allowed multipole is $E2$.

Problem 20.2: ^{117}In $1/2^- \to 9/2^+$

For $J_i^\pi = \frac{1}{2}^-$ to $J_f^\pi = \frac{9}{2}^+$, find allowed L values and the corresponding EL/ML assignments. Which multipole dominates? [6, 1]

Solution 20.2

Step 1:

$$\left|\tfrac{1}{2} - \tfrac{9}{2}\right| \le L \le \tfrac{1}{2} + \tfrac{9}{2} \quad \Rightarrow \quad L = 4, 5.$$

Step 2: Parity changes $(- \to +)$.

- $L = 4$: ML has $\pi_f = \pi_i(-1)^5 = -\pi_i \Rightarrow M4$. EL would preserve parity $\Rightarrow$ forbidden.
- $L = 5$: EL has $\pi_f = \pi_i(-1)^5 = -\pi_i \Rightarrow E5$. ML would preserve parity $\Rightarrow$ forbidden.

Step 3: Rate hierarchy $\lambda(L+1)/\lambda(L) \sim 10^{-5}$ and $\lambda(EL)/\lambda(ML) \sim 10^2$ still leave $M4$ far above $E5$. Dominant multipole: $M4$.

Problem 20.3: Why $0^+ \to 0^+$ cannot emit a photon

Show from the triangle inequality that a $0^+ \to 0^+$ transition cannot proceed by single-photon emission. What is the alternative electromagnetic channel? [6]

Solution 20.3

For $J_i = J_f = 0$,

$$|0-0| \le L \le 0+0 \quad \Rightarrow \quad L = 0.$$

A real photon multipole requires $L \geq 1$. Hence $\lambda_\gamma = 0$ for single-photon emission. The transition proceeds by internal conversion (electric monopole coupling to an atomic electron), or by e^+e^- pair creation if $E_{\text{tr}} > 1.022\,\text{MeV}$.

Problem 20.4: Conversion coefficient

A transition emits $N_\gamma = 8.0 \times 10^6$ photons and $N_e = 2.0 \times 10^6$ conversion electrons (all shells). Find α and the fraction of decays that proceed by photon emission [1].

Solution 20.4

$$\alpha = \frac{N_e}{N_\gamma} = \frac{2.0 \times 10^6}{8.0 \times 10^6} = 0.25.$$

Total electromagnetic decays $N_{\text{tot}} = N_\gamma + N_e = 1.0 \times 10^7$. Photon fraction:

$$\frac{N_\gamma}{N_{\text{tot}}} = \frac{1}{1+\alpha} = \frac{1}{1.25} = 0.80 \quad (80\%).$$

Problem 20.5: Recoil energy

Estimate the nuclear recoil energy $E_R = E_\gamma^2/(2Mc^2)$ for the 662 keV γ ray of ^{137}Ba ($Mc^2 \approx 137 \times 931\,\text{MeV}$). Is E_R large compared with eV-scale chemical shifts? [6]

Solution 20.5

$$Mc^2 \approx 1.275 \times 10^5\,\text{MeV}, \qquad E_\gamma = 0.662\,\text{MeV}.$$

$$E_R = \frac{(0.662)^2}{2 \times 1.275 \times 10^5}\,\text{MeV} = \frac{0.438}{2.55 \times 10^5}\,\text{MeV} \approx 1.72 \times 10^{-6}\,\text{MeV} = 1.72\,\text{eV}.$$

Yes: E_R is of order eV, comparable to or larger than many chemical and hyperfine shifts, which is why free-atom resonant absorption is hindered and the Mössbauer (recoilless) effect is powerful in solids.

Problem 20.6: Assign $3^- \to 2^+$

Determine the lowest multipole allowed for $3^- \to 2^+$ [6, 1].

Solution 20.6

Angular momentum:

$$|3-2| \leq L \leq 3+2 \quad \Rightarrow \quad L = 1, 2, 3, 4, 5.$$

Parity changes. Lowest $L = 1$:

$$E1: \ \pi_f = \pi_i(-1)^1 = -\pi_i \quad \text{(allowed)}, \qquad M1: \ \text{parity unchanged (forbidden)}.$$

Dominant multipole: $E1$. (Higher multipoles are strongly suppressed.)

Problem 20.7: Isomeric hindrance

Explain qualitatively why an $M4$ isomer can have a half-life of minutes while a typical $E2$ state lives for picoseconds [6, 14].

Solution 20.7

Each increase of multipole order by one unit suppresses the radiative width by roughly 10^{-5} (Weisskopf scale). Going from $L = 2$ ($E2$) to $L = 4$ ($M4$) costs about two steps of that factor, plus the extra $\lambda(EL)/\lambda(ML) \sim 10^2$ penalty for magnetic versus electric at fixed L. The net suppression can easily exceed 10^{10}, turning picosecond lifetimes into seconds or minutes. Selection rules that force high L are the origin of many nuclear isomers.

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