Chapter 1

Beta Decay: Modes, Q-Values, and the Neutrino

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

W. Pauli (1930), on the neutrino

Beta decay rearranges N and Z at fixed A through the weak interaction. When the neutron-to-proton ratio N/Z deviates from the value required for stability on the chart of nuclides, the nucleus can lower its mass by converting a neutron into a proton (β^- decay) or a proton into a neutron (β^+ decay or electron capture). This lecture develops the three decay modes with their full reaction equations (including neutrinos), derives Q-values first from nuclear masses and then from tabulated atomic masses (showing every electron term until it cancels), compares the energetics of β^+ and electron capture, and explains the continuous β spectrum that forced Pauli's neutrino hypothesis and Fermi's theory of β decay. The exposition follows Krane, Dunlap, and Demtröder [6, 1, 2, 5].

1.1 When the N/Z ratio is wrong

A stable nucleus requires a proper balance of neutrons and protons. For light nuclei the stable N/Z ratio is near unity; for heavy nuclei it rises toward ~ 1.5 because the Coulomb repulsion among protons must be compensated by extra neutrons [6, 1]. Two generic failures illustrate the logic:

1. **Excess of neutrons.** Consider $Z = 10$ with $N = 14$ on the stability line. If N increases to 16 while Z stays fixed, the N/Z ratio moves away from stability. A neutron inside the nucleus can convert to a proton, increasing Z by one and decreasing N by one. This is β^- decay.
2. **Excess of protons.** With the same $Z = 10$, $N = 14$ reference, suppose Z increases to 12 while N remains 14. Even though neutrons still outnumber protons, the proton count exceeds the stable value for this mass number. A proton converts to a neutron, decreasing Z by one. This is β^+ decay or electron capture.

Both conversions are mediated by the weak interaction. At the quark level the fundamental processes are

$$n \to p + e^- + \bar{\nu}_e, \qquad p \to n + e^+ + \nu_e, \qquad p + e^- \to n + \nu_e. \tag{1.1}$$

Electrons and positrons are not pre-existing constituents of the nucleus; they are created at the instant of decay, in contrast with α particles in α decay [6, 2].

1.2 Three modes of beta decay

For a parent nucleus ${}^A_Z X$ the three observed modes are:

Beta minus (β^-).

$${}^A_Z X \ \to \ {}^{A}_{Z+1} Y + e^- + \bar{\nu}_e. \tag{1.2}$$

The proton number increases by one; a neutron becomes a proton. The free neutron decays by this mode with half-life $t_{1/2} \approx 879\,\mathrm{s}$ and $Q \approx 0.782\,\mathrm{MeV}$ [6, 20].

Beta plus (β^+).

$${}^A_Z X \ \to \ {}^{A}_{Z-1} Y + e^+ + \nu_e. \tag{1.3}$$

The proton number decreases by one. A *free* proton cannot decay this way because $m_n > m_p$ and the two-body final state would violate energy conservation. Inside a proton-rich nucleus, however, the change in nuclear binding energy can make the process exothermic. Examples include ^{11}C and ^{18}F, used in positron-emission tomography [6, 1, 21].

Electron capture (EC).

$${}^A_Z X + e^- \ \to \ {}^{A}_{Z-1} Y + \nu_e. \tag{1.4}$$

The nucleus captures an atomic electron (usually from the K shell), converting a proton to a neutron. The daughter atom is left with a vacancy that fills by characteristic X-rays or Auger electrons. EC and β^+ both reduce Z by one and convert a proton to a neutron [6, 1].

Key idea

β^-: $Z \to Z+1$, antineutrino $\bar{\nu}_e$ emitted. β^+ and EC: $Z \to Z-1$, neutrino ν_e emitted. Free neutron decays; free proton does not. All three modes require a light, neutral, spin-$\frac{1}{2}$ neutrino (or antineutrino) to conserve energy, momentum, and angular momentum [6, 2].

Historical note

In December 1930 Wolfgang Pauli proposed an undetected neutral particle to save energy and angular momentum in β decay. Enrico Fermi named it the *neutrino* and built the first quantitative theory in 1934 [6, 2, 5]. Direct detection came only in 1956 (Reines and Cowan); today we distinguish ν_e from $\bar{\nu}_e$ in each decay mode as written above.

1.3 Q-value for β^-: nuclear masses, then atomic masses

Consider reaction (1.2). The Q-value is the difference between initial and final rest mass energies (excluding the negligible nuclear recoil):

$$Q_{\beta^-} = \Big[m'({}^A_Z X) - m'({}^{A}_{Z+1}Y) - m_e - m(\bar{\nu}_e)\Big]c^2, \tag{1.5}$$

where $m'(\cdot)$ denotes *nuclear* mass (nucleons only). The antineutrino mass is of order eV and is neglected: $m(\bar{\nu}_e) \approx 0$.

Step 1: express nuclear masses through atomic masses. Tables list *atomic* masses $m(\cdot)$ of neutral atoms. A neutral atom ${}^A_Z X$ contains the nucleus plus Z orbital electrons. Neglecting electronic binding energies B_i (eV scale, compared with MeV nuclear Q-values),

$$m'({}^A_Z X) = m({}^A_Z X) - Zm_e. \tag{1.6}$$

For the daughter ${}^{A}_{Z+1}Y$, which has $Z+1$ electrons in its neutral atom,

$$m'({}^{A}_{Z+1}Y) = m({}^{A}_{Z+1}Y) - (Z+1)m_e. \tag{1.7}$$

Step 2: substitute into (1.5)**.**

$$\begin{aligned} Q_{\beta^-} &= \Big[(m(X) - Zm_e) - (m(Y) - (Z+1)m_e) - m_e\Big]c^2 \\ &= \Big[m(X) - Zm_e - m(Y) + (Z+1)m_e - m_e\Big]c^2. \end{aligned} \tag{1.8}$$

Step 3: collect electron terms. The electron mass terms are

$$-Zm_e + (Z+1)m_e - m_e = -Zm_e + Zm_e + m_e - m_e = 0. \tag{1.9}$$

Every explicit electron mass cancels. Therefore

$$Q_{\beta^-} = \big[m({}^A_Z X) - m({}^{A}_{Z+1}Y)\big]c^2, \tag{1.10}$$

using tabulated atomic masses directly [6, 1]. The energy Q_{β^-} is shared by the electron, antineutrino, and (negligibly) the recoiling daughter:

$$Q_{\beta^-} = K_e + E_\nu, \qquad K_{\max} = Q_{\beta^-} \quad (\text{when } E_\nu \to 0). \tag{1.11}$$

Remark

Small differences in electronic binding between parent and daughter atoms (typically keV) are omitted above. Precision work sometimes retains them; for course estimates they are negligible compared with $Q \sim 1$–$3\,\text{MeV}$ [6].

1.4 Q-value for β^+: the $2m_ec^2$ threshold

For reaction (1.3), with nuclear masses and $m(\nu_e) \approx 0$,

$$Q_{\beta^+} = \Big[m'(X) - m'(Y) - m_e\Big]c^2, \tag{1.12}$$

where m_e is the positron mass ($m_{e^+} = m_{e^-}$). Using (1.6) with $m'(Y) = m(Y) - (Z-1)m_e$,

$$\begin{aligned} Q_{\beta^+} &= \Big[(m(X) - Zm_e) - (m(Y) - (Z-1)m_e) - m_e\Big]c^2 \\ &= \Big[m(X) - Zm_e - m(Y) + (Z-1)m_e - m_e\Big]c^2. \end{aligned} \tag{1.13}$$

Collecting electron terms:

$$-Zm_e + (Z-1)m_e - m_e = -Zm_e + Zm_e - m_e - m_e = -2m_e. \tag{1.14}$$

Hence

$$Q_{\beta^+} = [m(X) - m(Y)]c^2 - 2m_ec^2. \tag{1.15}$$

Positron emission requires not only $m(X) > m(Y)$ but

$$m(X) - m(Y) > 2m_e \quad \Longleftrightarrow \quad Q_{\beta^+} > 0, \tag{1.16}$$

with threshold $2m_ec^2 = 1.022\,\text{MeV}$. Typical β Q-values are only a few MeV, so this threshold is a substantial fraction of the available energy [6, 1].

1.5 Q-value for electron capture

For reaction (1.4), the captured electron is a reactant:

$$Q_{\text{EC}} = \Big[m'(X) + m_e - m'(Y)\Big]c^2 - B_e, \tag{1.17}$$

where B_e is the binding energy of the captured electron in the parent atom. Converting to atomic masses,

$$\begin{aligned} Q_{\text{EC}} &= \Big[(m(X) - Zm_e) + m_e - (m(Y) - (Z-1)m_e)\Big]c^2 - B_e \\ &= [m(X) - m(Y)]c^2 - B_e. \end{aligned} \tag{1.18}$$

All electron *mass* terms cancel exactly as in β^-; unlike β^+, no positron rest energy must be supplied. The captured electron's *binding* energy B_e is subtracted because that energy is already accounted for in the atomic mass of the parent.

Caution

B_e is *not* universally a few eV. For K-shell capture in medium and heavy elements, B_e can be tens of keV (for example $B_K \approx 80\,\text{keV}$ for iodine). Always use the binding energy of the shell from which capture occurs [6, 1].

Comparison of β^+ and EC. Subtracting (1.15) from the EC result (neglecting B_e for a rough estimate),

$$Q_{\mathrm{EC}} - Q_{\beta^+} \approx 2m_e c^2 = 1.022\,\mathrm{MeV}. \tag{1.19}$$

Whenever

$$0 < [m(X) - m(Y)]c^2 < 2m_e c^2, \tag{1.20}$$

electron capture is energetically allowed ($Q_{\mathrm{EC}} > 0$) but positron emission is forbidden ($Q_{\beta^+} < 0$). This situation is common for moderately proton-rich nuclei [6, 1, 20].

Key idea

Atomic-mass formulas (neglecting B_e): $Q_{\beta^-} = [m(X) - m(Y)]c^2$, $Q_{\beta^+} = [m(X) - m(Y) - 2m_e]c^2$, $Q_{\mathrm{EC}} = [m(X) - m(Y)]c^2 - B_e$. EC is favored over β^+ by $\approx 2m_e c^2$.

1.6 Continuous spectrum: ^{64}Cu and the missing energy

The Q-value appears as kinetic energy shared among the light decay products. The recoiling daughter nucleus carries negligible energy ($\sim$ keV or less). For β^- decay the electron and antineutrino share Q_{β^-}; neither body is at rest in general, so the electron kinetic energy K_e is *continuous* from near zero up to $K_{\max} \approx Q$.

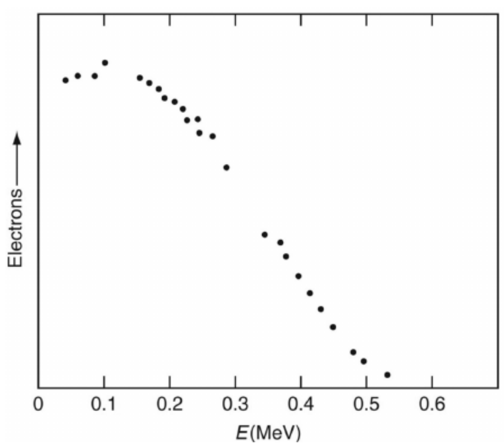

Figure 1.1: Energy spectrum of electrons from β^- decay of ^{64}Cu to ^{64}Zn. The endpoint near 0.58 MeV equals Q_{β^-} for the ground-state transition. Data from Langer, Moffat, and Price [29], as reproduced in Dunlap [1, 6].

Copper-64 is a remarkable laboratory example: it decays by both β^- and β^+ (and EC), so both spectra in Figs. 1.1 and 1.2 can be measured on one isotope [1, 21].

In the 1920s, before the neutrino was known, a *discrete* electron line was expected. If only the electron and heavy daughter emerged, two-body kinematics would fix the electron energy (as in α decay). Calorimetric experiments showed that the continuous distribution is intrinsic to the decay, not caused by energy loss in the source [6, 2]. Electrons were often observed with kinetic energy well below Q: where was the missing energy?

Pauli's solution (1930): a second, electrically neutral, highly penetrating particle carries away the "missing" energy and spin. Fermi (1934) incorporated the neutrino into a quantitative

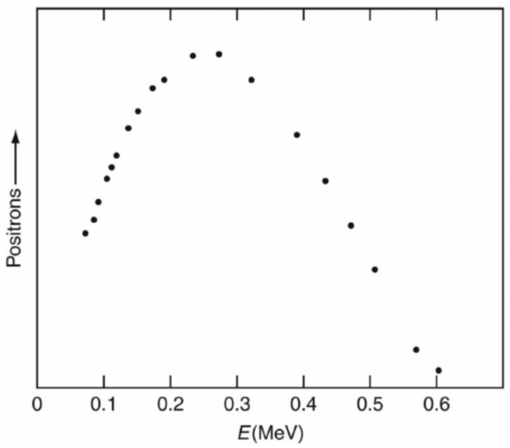

Figure 1.2: Energy spectrum of positrons from β^+ decay of ^{64}Cu to ^{64}Ni. Endpoint near 0.65 MeV. Same experimental reference as Fig. 1.1 [29, 1, 6].

theory. The continuous spectrum is therefore a *three-body* phase-space effect, not an experimental artefact [6, 2, 5].

Caution

The continuous spectrum is three-body kinematics, not instrumental smearing. The endpoint energy measures Q (and, in precision experiments near the endpoint, constrains the neutrino mass; see Lecture 19). Internal conversion and atomic effects can add low-energy structure, but the broad envelope is phase space [6, 14].

1.7 Toward Fermi theory: golden rule and plane-wave leptons

Fermi treated the weak interaction as a small perturbation on nuclear states. The transition rate from initial state ψ_i to final state ψ_f is Fermi's golden rule [6, 1, 14]:

$$\lambda = \frac{2\pi}{\hbar} |\langle \psi_f | H_{\text{int}} | \psi_i \rangle|^2 \frac{\mathrm{d}n}{\mathrm{d}E_f}, \tag{1.21}$$

where $\mathrm{d}n/\mathrm{d}E_f$ is the density of final states. For β^- decay,

$$\psi_i = \psi_p, \qquad \psi_f = \psi_d \, \psi_e \, \psi_\nu, \tag{1.22}$$

with ψ_p and ψ_d the parent and daughter nuclear wave functions.

The electron and antineutrino are treated as free plane waves normalized in volume V:

$$\psi_e(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{p}_e \cdot \boldsymbol{r}/\hbar} = \frac{1}{\sqrt{V}} \left(1 + \frac{i\boldsymbol{p}_e \cdot \boldsymbol{r}}{\hbar} + \cdots \right), \tag{1.23}$$

and similarly for ψ_ν. For typical β momenta ($p \sim 1\,\text{MeV}/c$) and nuclear radius $R \sim 5\,\text{fm}$, the dimensionless parameter $pR/\hbar \sim 0.03 \ll 1$. The *allowed* approximation keeps only the constant term ($\ell = 0$ for the lepton pair). The nuclear physics is then absorbed into a matrix element M_{fi} nearly independent of lepton energy, and the spectrum shape comes entirely from the lepton phase space developed in Lecture 19 [6, 1].

Key idea

Students should remember:

- Wrong $N/Z \Rightarrow \beta^-$ (neutron rich) or β^+/EC (proton rich).
- Free neutron decays; free proton does not ($m_n > m_p$).
- Atomic-mass $Q_{\beta^-} = [m(X) - m(Y)]c^2$; all electron masses cancel.
- Q_{β^+} has an extra $-2m_e c^2$; EC is $\approx 1.022\,\text{MeV}$ more favorable.
- Continuous spectrum $\Rightarrow$ three bodies $\Rightarrow$ neutrino.
- Golden rule: rate $\propto |M_{fi}|^2 \times$ (phase space).

1.8 Further physics from the literature

Free neutron decay. The simplest β^- example is free neutron decay, $n \to p + e^- + \bar{\nu}_e$, with measured endpoint $K_{\max} = 0.782\,\text{MeV}$. This value fixes the neutron–proton mass difference and provides one of the most precise tests of the V-A structure of the weak interaction [6, 2, 20].

Double beta decay. When single β decay is blocked (for example, when the intermediate nucleus would have higher mass), some even–even nuclei can decay by double β decay. Two-neutrino double β ($2\nu\beta\beta$) has been observed; neutrinoless double β ($0\nu\beta\beta$), if found, would prove that neutrinos are Majorana particles and would constrain the absolute neutrino mass scale [6, 1, 14, 5].

Parity violation. The Wu experiment (1957) showed that β decay does not conserve parity: electrons are preferentially emitted opposite to the nuclear spin in polarised ^{60}Co decay. This established the V-A form of the weak current and explained why only left-handed neutrinos and right-handed antineutrinos appear in β decay at low energy [2, 5, 6].

Medical and geochemical applications. Positron emitters (^{11}C, ^{18}F) and β^- sources are used in PET and radiotherapy. The β^- decay of ^{14}C provides radiocarbon dating. EC of ^{40}K contributes to the Earth's internal heat [6, 1, 13].

From the literature

Dunlap emphasises that calorimetric measurements on ^{210}Bi proved the continuous spectrum is a property of the decay electrons themselves, not energy loss before detection [1]. Krane's Table 9.1 lists representative β^-, β^+, and EC processes with Q and half-lives for order-of-magnitude estimates [6].

1.9 Problems with solutions

Problem 18.1 — Derive Q_{β^-} from atomic masses

Show step by step that for ${}^A_Z X \to {}^A_{Z+1}Y + e^- + \bar{\nu}_e$,

$$Q_{\beta^-} = [m({}^A_Z X) - m({}^A_{Z+1}Y)]c^2, \tag{1.24}$$

starting from nuclear masses and neglecting $m(\bar{\nu}_e)$ and electronic binding [6].

Step-by-step

Step 1. Write the nuclear-mass Q-value:

$$Q_{\beta^-} = [m'(X) - m'(Y) - m_e]c^2.$$

Step 2. Relate nuclear and atomic masses:

$$m'(X) = m(X) - Zm_e, \qquad m'(Y) = m(Y) - (Z+1)m_e.$$

Step 3. Substitute:

$$Q_{\beta^-} = [m(X) - Zm_e - m(Y) + (Z+1)m_e - m_e]c^2.$$

Step 4. Simplify the electron terms:

$$-Zm_e + (Z+1)m_e - m_e = (-Z + Z + 1 - 1)m_e = 0.$$

Step 5. Conclude $Q_{\beta^-} = [m(X) - m(Y)]c^2$. All three electron masses (one in the daughter atom, one emitted, one subtracted explicitly) cancel identically [6, 1].

Problem 18.2 — β^+ threshold and EC window

The atomic mass excess of a parent exceeds that of the daughter by $\Delta m\, c^2 = 1.50\,\text{MeV}$ (parent heavier). Can the nucleus decay by β^+? By EC from the K shell with $B_K = 0$? What if $\Delta m\, c^2 = 0.80\,\text{MeV}$?

Step-by-step

Case A: $\Delta m\, c^2 = 1.50\,\text{MeV}$.
$Q_{\beta^+} = 1.50 - 1.022 = 0.48\,\text{MeV} > 0$: positron emission is allowed.
$Q_{\text{EC}} = 1.50 - B_K = 1.50\,\text{MeV} > 0$: EC is also allowed. Both modes compete; EC has the larger Q by $2m_ec^2$ [6, 1].
Case B: $\Delta m\, c^2 = 0.80\,\text{MeV}$.
$Q_{\beta^+} = 0.80 - 1.022 = -0.22\,\text{MeV} < 0$: positron emission is forbidden.
$Q_{\text{EC}} = 0.80\,\text{MeV} > 0$: only EC (and any other exothermic mode) is open. This illustrates the window $0 < \Delta m\, c^2 < 2m_ec^2$ where EC alone occurs [1, 20].

Problem 18.3 — ^{64}Cu endpoint

Using atomic mass tables, estimate Q_{β^-} for $^{64}\mathrm{Cu} \to {}^{64}\mathrm{Zn} + e^- + \bar{\nu}_e$ and compare with the endpoint in Fig. 1.1.

Step-by-step

From ENSDF/NuDat [20, 21], the ground-state Q_{β^-} for ^{64}Cu is 0.579 MeV. Dunlap quotes 0.579 MeV from atomic masses, in agreement with the Langer *et al.* spectrum endpoint near 0.58 MeV (Fig. 1.1) [1, 6]. Electrons with $K = Q$ are emitted with $E_{\bar{\nu}} \to 0$; the spectrum falls to zero at this endpoint because the $(Q - K)^2$ phase-space factor vanishes (derived fully in Lecture 19).

Problem 18.4 — Why is the spectrum continuous?

Explain in three sentences why α decay gives discrete α energies but β^- decay gives a continuous electron spectrum, even for a ground-state to ground-state transition.

Step-by-step

α decay is a two-body process (α + daughter nucleus). Energy and momentum conservation fix the α kinetic energy uniquely (up to small recoil corrections). β^- decay is a three-body process (electron + antineutrino + daughter). One continuous parameter (for example the antineutrino energy) remains undetermined, so the electron kinetic energy ranges continuously from 0 to Q [6, 2].

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