Chapter 1

Theories of Nuclear Forces

"Yukawa noticed that the range of nuclear forces $r_0 \simeq 1.4\,\mathrm{fm}$ *corresponds to a particle of mass* $m_\pi c^2 \simeq 140\,\mathrm{MeV}$*."*

paraphrasing H. Yukawa (1935)

Lectures 7–10 constrain the residual NN force from two sides: the deuteron multipoles and low-energy phase shifts. Both approaches are phenomenological: they fix a potential (or its phase shifts) to data. This lecture asks a deeper question: *what carries the force?* In modern quantum field theory, forces between particles arise from the exchange of virtual quanta. For electromagnetism the quantum is the massless photon and the potential is Coulomb. For the residual strong force the lightest quantum is the massive pion and the potential is Yukawa [9, 14, 5, 15, 2].

1.1 Three layers of description: from Coulomb to QED

Electromagnetism shows how the same interaction can be told at increasing depth.

Action at a distance. Two charges q_1 and q_2 exert equal and opposite forces

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \tag{1.1}$$

instantaneously across empty space. The formula works at low speeds, but instantaneous action conflicts with relativity: no influence can travel faster than light.

Classical field. Charge q_1 does not "reach out" to q_2. It produces an electric field filling space; q_2 feels a force because it sits in that field. The field is the intermediary. Changes in the field propagate at finite speed c.

Quantum field (QED). The electromagnetic field is quantized. Energy and momentum are exchanged in discrete packets (photons). The Coulomb force is the low-energy limit of exchanging *virtual* photons: off-shell quanta that mediate the interaction without appearing as free radiation. Virtual photons share the quantum numbers of real photons (massless, chargeless, spin 1); only their four-momentum differs from the on-shell condition $q^2 = 0$ [5, 15].

The same ladder applies to nucleons N_1 and N_2. Classically one might imagine a potential $V(r)$. At the quantum level the interaction is the exchange of virtual mesons. Unlike the photon, those mesons may carry mass and electric charge, and that changes the range and the selection rules of the force.

1.2 Range of a force from the uncertainty principle

A useful *heuristic* estimates the range of a force mediated by a massive quantum. Creating a virtual particle of rest energy mc^2 requires an energy fluctuation of order $\Delta E \sim mc^2$. The uncertainty principle then suggests a lifetime

$$\Delta t \lesssim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{mc^2}. \tag{1.2}$$

Even if the quantum travels near the speed of light, the farthest it can go before being reabsorbed is of order

$$R \lesssim c\,\Delta t \sim \frac{\hbar c}{mc^2} = \frac{\hbar}{mc}. \tag{1.3}$$

This Compton wavelength of the mediator sets the force range. Two limits are immediate:

- **Massless mediator** ($m = 0$): $R \to \infty$. The force is long-ranged (Coulomb, gravity in the Newtonian limit).
- **Massive mediator**: R is finite. The force is short-ranged; beyond a few Compton wavelengths the exchange is exponentially suppressed.

This argument is pedagogical, not a derivation of field theory: it does not claim that energy conservation is violated. The rigorous statement is that a massive propagator $1/(q^2 + m^2c^2)$ Fourier-transforms to an exponentially screened potential, derived next from the Klein–Gordon equation [9, 14, 5, 2].

That single scale explains why nuclear forces die within a few fm while electrostatics still acts across atomic distances.

1.2.1 Predicting the pion mass from the nuclear range

Electron scattering and low-energy NN data fix the range of the residual strong force at $R \sim 1$–$2\,\mathrm{fm}$. Taking $R \approx 1.4$–$2\,\mathrm{fm}$ and $\hbar c \approx 197\,\mathrm{MeV\,fm}$,

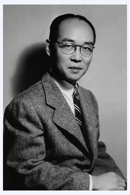
$$mc^2 \sim \frac{\hbar c}{R} \approx \frac{197}{1.4\text{–}2} \approx 100\text{–}140\,\mathrm{MeV}. \tag{1.4}$$

Yukawa made this prediction in 1935, years before the pion was discovered. The observed mesons have

$$m_{\pi^\pm}c^2 \approx 139.6\,\mathrm{MeV}, \qquad m_{\pi^0}c^2 \approx 135\,\mathrm{MeV},$$

exactly the scale required by the nuclear force range [9, 14, 2].

Historical note: Hideki Yukawa (1907–1981)



Hideki Yukawa, working in Osaka in 1934–1935, proposed that the short-range nuclear force is mediated by the exchange of a massive boson. From the empirical range $r_0 \sim 1$–$2\,\mathrm{fm}$ he estimated a mediator mass of order $100\,\mathrm{MeV}/c^2$. The particle, the pion, was later found with $m_\pi c^2 \approx 140\,\mathrm{MeV}$, and the associated potential falls exponentially with range fixed by m_π. Yukawa received the 1949 Nobel Prize in Physics, the first Nobel awarded to a Japanese scientist. His insight is the microscopic origin of the residual NN force developed in this lecture [9, 14, 2].

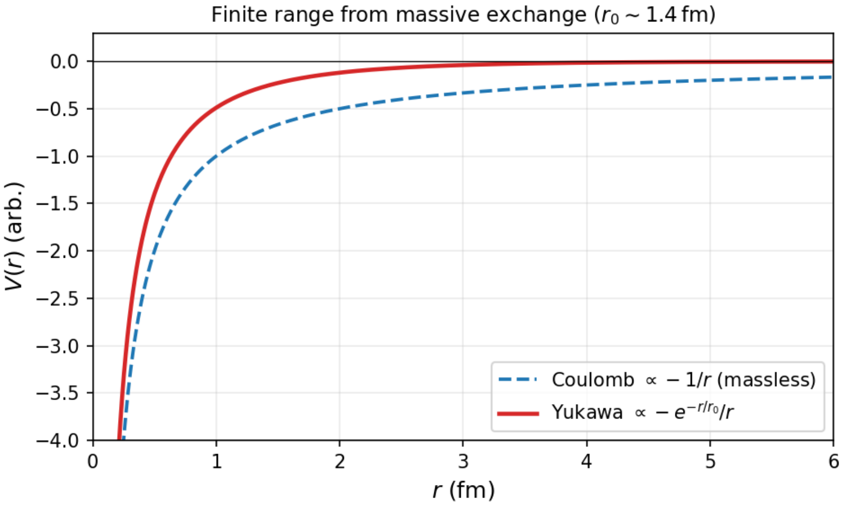


Figure 1.1: Coulomb ($-1/r$, massless exchange) versus Yukawa ($-e^{-r/r_0}/r$, massive exchange). The exponential kills the force beyond $r_0 \sim \hbar/(m_\pi c)$.

Key idea

Finite nuclear range $\Leftrightarrow$ massive exchange quantum. The pion mass and the fermi-scale range of the residual NN force are two faces of $R \sim \hbar/(m_\pi c)$.

1.3 From the Klein–Gordon equation to the Yukawa potential

The form of the potential follows from the relativistic wave equation for a spinless field of mass m. The free Klein–Gordon equation is

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \mu^2\psi = 0, \qquad \mu = \frac{mc}{\hbar}. \tag{1.5}$$

For $m = 0$ this reduces to the wave equation for electromagnetic potentials. A static point source for the massless case yields Coulomb's law. With mass, a static isotropic source at the origin obeys

$$(\nabla^2 - \mu^2)V(r) = -g\,\delta^3(\boldsymbol{r}). \tag{1.6}$$

For $r \neq 0$ the radial equation is

$$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(rV) - \mu^2 V = 0,$$

so $rV = Ae^{-\mu r} + Be^{+\mu r}$. Boundedness at infinity forces $B = 0$, hence $V(r) = Ae^{-\mu r}/r$. To fix A, integrate (1.6) over a ball of radius $\varepsilon \to 0$:

$$\int_{r<\varepsilon} (\nabla^2 - \mu^2) V \, \mathrm{d}^3 r = \oint_{\partial B} \nabla V \cdot \mathrm{d}\boldsymbol{S} - \mu^2 \int V \, \mathrm{d}^3 r = -g. \tag{1.7}$$

The volume integral of $V \sim 1/r$ vanishes as $\varepsilon \to 0$, while $\nabla V \sim -A\hat{r}/r^2$ gives $\oint \nabla V \cdot \mathrm{d}\boldsymbol{S} = -4\pi A$. Therefore $A = g/(4\pi)$ (up to the overall coupling convention), and

$$V(r) = -\frac{g^2}{4\pi} \frac{\hbar c}{r} e^{-\mu r} = -\frac{g^2}{4\pi} \frac{\hbar c}{r} e^{-r/r_0}, \qquad r_0 = \frac{\hbar}{mc} \approx 1.4\,\mathrm{fm} \quad (\text{for } m = m_\pi). \tag{1.8}$$

(The overall sign and coupling convention depend on the channel; the attractive form shown here is the pedagogical scalar Yukawa well. Physical OPEP has spin–isospin structure and a tensor piece.)

Fourier-transform route. The same potential is the Fourier transform of the massive propagator. In the static Born approximation,

$$\tilde{V}(\boldsymbol{q}) = -\frac{g^2 \hbar c}{q^2 + \mu^2}, \tag{1.9}$$

and

$$\begin{aligned} V(r) &= \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \tilde{V}(\boldsymbol{q}) \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} = -\frac{g^2 \hbar c}{(2\pi)^2} \int_0^\infty \frac{q^2 \, \mathrm{d}q}{q^2 + \mu^2} \frac{\sin(qr)}{qr} \\ &= -\frac{g^2 \hbar c}{4\pi} \frac{e^{-\mu r}}{r}, \end{aligned} \tag{1.10}$$

recovering (1.8). In the limit $m \to 0$ ($\mu \to 0$) one recovers the Coulomb shape $1/r$; for finite m the exponential cuts off the force beyond r_0 [14, 5].

In quantum field theory the same potential is the Fourier transform of the propagator $1/(q^2 + m^2 c^2)$ of the exchanged meson in the static Born approximation [14, 5].

1.4 Pions as mediators of the residual strong force

Real pions are copiously produced in accelerators and in cosmic-ray showers in the atmosphere. Virtual pions of the same mass and quantum numbers can be exchanged between nucleons; the associated range is their Compton wavelength $\hbar/(m_\pi c) \sim 1.4\,\mathrm{fm}$.

It is crucial to distinguish the **fundamental** strong force from the **residual** force between nucleons. Inside a nucleon, quarks exchange gluons and the interaction is extremely strong and confining. A proton or neutron as a whole is a colour singlet (“white”). When two colour-neutral nucleons approach within a fermi, residual colour forces remain, analogous to van der Waals forces between neutral atoms. Pion exchange is the longest-range piece of that residual interaction [15, 4, 14].

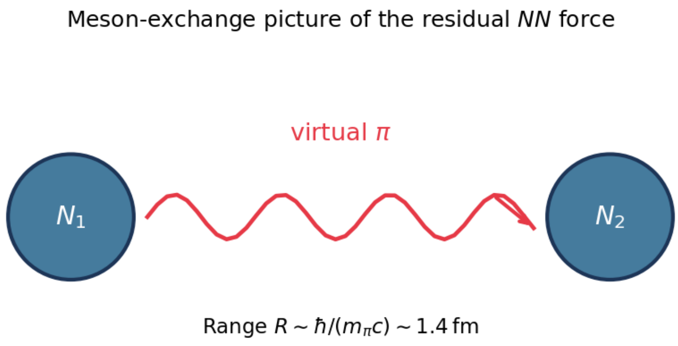


Figure 1.2: Schematic meson exchange between two nucleons. The virtual pion mediates a short-range residual force of Compton wavelength $\hbar/(m_\pi c)$.

1.4.1 Charge exchange: pp versus np

Neutral pion exchange preserves nucleon identity:

$$p \to p + \pi^0, \qquad n \to n + \pi^0. \tag{1.11}$$

Charged pion exchange converts neutron $\leftrightarrow$ proton:

$$n \to p + \pi^-, \qquad p + \pi^- \to n, \tag{1.12}$$
$$p \to n + \pi^+, \qquad n + \pi^+ \to p. \tag{1.13}$$

Consequently:

- In pp (or nn) scattering, if both nucleons keep their charges, only π^0 exchange is allowed at the one-pion level.
- In np scattering, π^0, π^+, and π^- can all contribute.

Because $m_{\pi^\pm}$ and m_{π^0} differ by only a few MeV, the pp and np forces are nearly equal once the Coulomb repulsion between protons is removed. That near-equality is the charge independence of the residual strong force, already used when arguing that only the pn triplet binds (Lecture 8) [14, 6].

1.5 One-pion exchange, tensor force, and the short-range core

Single-pion exchange (OPEP) produces not only the radial Yukawa factor (1.8) but also spin-dependent and tensor structures. The pseudoscalar πNN coupling leads, in the static limit, to a momentum-space amplitude proportional to

$$\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{q^2 + m_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2.$$

Fourier transforming and rearranging with the identity

$$(\boldsymbol{\sigma}_1 \cdot \hat{q})(\boldsymbol{\sigma}_2 \cdot \hat{q}) = \tfrac{1}{3}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \tfrac{1}{3}S_{12}(\hat{q}) \tag{1.14}$$

separates a central spin–spin piece from the tensor operator

$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{r})(\boldsymbol{\sigma}_2 \cdot \hat{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2. \tag{1.15}$$

Differentiating the Yukawa function twice (the coordinate-space image of $q_i q_j/(q^2+m_\pi^2)$) produces the standard radial factors

$$V_C^\pi(r) \propto \frac{e^{-m_\pi r}}{r}, \tag{1.16}$$

$$V_T^\pi(r) \propto \frac{e^{-m_\pi r}}{r}\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right). \tag{1.17}$$

The potential piece $V_T(r)S_{12}$ is attractive in the deuteron channel and is the long-range origin of the S–D mixing, the quadrupole moment $Q_d \neq 0$, and the small correction to μ_d discussed in Lecture 9 [14, 6]. In multi-nucleon systems the tensor force largely averages out; in the two-body problem it is essential for binding.

A pure attractive Yukawa well is not enough for nuclear saturation. Phenomenological and meson-exchange potentials add a strong **repulsive core** at $r \lesssim 0.5\,\mathrm{fm}$, often modelled as another Yukawa with a heavier mass (or as ω and multi-pion exchange). Heavier mediators have shorter Compton wavelengths, so they act only at short distance, exactly where the hard core is needed to keep nucleons from collapsing into a single blob of pion-range size [14, 4].

A realistic NN potential therefore has a hierarchy of ranges [14]:

Distance	Dominant physics	Mediators / language
$r \gtrsim 1.4\,\mathrm{fm}$	long-range attraction, tensor	π (OPEP)
$r \sim 0.7$–$1.4\,\mathrm{fm}$	intermediate attraction	multi-π, ρ
$r \lesssim 0.5\,\mathrm{fm}$	short-range repulsion	ω, quark Pauli / contact terms in EFT

Modern chiral effective field theory organises the same hierarchy as an expansion in soft momenta: one-pion exchange at long range, multi-pion exchange at intermediate range, and short-range contact operators fitted to phase shifts. For this course it is enough that OPEP explains the long-range tensor force while shorter-range physics is constrained by scattering data (Lecture 10), not invented from the deuteron alone.

1.5.1 Phenomenology and mechanism

Historically, NN potentials were first fitted to deuteron properties and scattering phase shifts with adjustable strengths and ranges (phenomenology). Yukawa's insight and modern one-boson-exchange models derive the long-range form from a field-theoretic mechanism. The two approaches meet in practice: phase shifts (Lecture 10) and the deuteron multipoles (Lectures 7–9) remain the data that any mechanism-based potential must reproduce [6, 14].

Observable (L7–10)	Inferred operator	Mechanism (this lecture)
B_d, no excited states	depth/range of attraction	residual force scale
$I^\pi = 1^+$, triplet binds	spin dependence	$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$Q_d \neq 0$, μ_d deficit	tensor force S_{12}	OPEP tensor
$a_t > 0$, $a_s < 0$, $\delta_\ell(E)$	channel-dependent V	OPE + shorter range

Weak interactions provide a contrast. Exchange of the heavy Z^0 ($m_Z c^2 \approx 91\,\text{GeV}$) yields a Yukawa range $\hbar c/m_Z c^2 \sim 2\times10^{-3}\,\text{fm}$ and an effective $V_0 R^2$ five orders of magnitude smaller than the strong residual force, negligible for nuclear binding though essential for neutrino scattering [14].

Key idea

Lectures 7–11 in one line. The deuteron and phase shifts map the residual NN potential; pion exchange explains why that potential is short-ranged, spin- and tensor-dependent, and charge-independent at long distance.

1.6 Further physics: residual force and colour neutrality

Nucleons are colour singlets. The fundamental strong interaction binds quarks via gluon exchange and confines colour. The force between two nucleons is a residual, colour-van-der-Waals effect: short-ranged, saturating, and much weaker than the force inside a single nucleon [15, 4, 14]. Pion exchange is the longest piece of that residual force; heavier mesons and quark substructure dominate only at sub-fermi distances where the repulsive core appears.

1.7 Problems with solutions

Problem 11.1 — Pion mass from range

If the nuclear force range is $R = 1.4\,\text{fm}$ and $\hbar c = 197\,\text{MeV fm}$, estimate the mediator mass $mc^2 \approx \hbar c/R$. Compare with $m_\pi c^2 \approx 140\,\text{MeV}$.

Solution 11.1

$$mc^2 \approx \frac{197}{1.4} \approx 141\,\text{MeV},$$

in excellent agreement with the pion mass [9, 14].

Problem 11.2 — Photon vs pion range

Why does electromagnetism have infinite range while the residual strong force does not, in the exchange picture?

Solution 11.2

The photon is massless, so Δt can be arbitrarily long and $R \sim c\Delta t$ unbounded. The pion is massive, so $\Delta t \lesssim \hbar/(m_\pi c^2)$ and $R \sim \hbar/(m_\pi c)$ is a few fm [5, 9].

Problem 11.3 — Which pions in pp vs np?

Which pion species can mediate pp scattering if both protons remain protons? Which can mediate np?

Solution 11.3

pp: only π^0 (charged emission would turn a proton into a neutron). np: π^0, π^+, and π^- all allowed via charge-exchange and neutral channels [14].

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