Chapter 1

The Deuteron and Nuclear Force

"For nuclear physicists, the deuteron should be what the hydrogen atom is for atomic physicists."

K. S. Krane

1.1 Why the deuteron is special

A deuteron is a bound proton–neutron system: the nucleus of deuterium (^{2}H). It is the simplest bound state of nucleons, and therefore an ideal laboratory for the nucleon–nucleon (NN) interaction.

In atomic physics, the hydrogen spectrum taught the Coulomb force and quantum electrodynamics. In nuclear physics one would like a similar study for the deuteron: measure transitions between excited states and read off the force. **Unfortunately there are no bound excited states of the deuteron.** The system is so weakly bound that any excitation puts the proton and neutron into the continuum. All information must therefore come from the ground-state binding energy and size, from ground-state multipoles (spin, magnetic moment, quadrupole moment), and from scattering of free nucleons [6, 2].

Key idea

The deuteron is the hydrogen atom of nuclear physics, with one crucial difference: it has only a ground state. Every number measured is a property of that single state, so the force must be extracted carefully and honestly.

1.2 Binding energy: three independent measurements

The binding energy B_d is known to high precision. Three independent methods agree [6].

1.2.1 Definition

Working always with *atomic* masses (as in Lectures 4–5),

$$B_d = [m(^1\mathrm{H}) + m(n) - m(^2\mathrm{H})]c^2. \tag{1.1}$$

With $c^2 = 931.50\,\mathrm{MeV/u}$, mass differences in atomic mass units convert directly to MeV.

1.2.2 Method 1: mass spectrometry

Mass doublets involving carbon and deuterium compounds fix $m(^2\mathrm{H})$ to better than $10^{-8}\,\mathrm{u}$. Combined with the hydrogen and neutron masses, one finds

$$B_d = 2.22463 \pm 0.00004\,\mathrm{MeV}. \tag{1.2}$$

1.2.3 Method 2: radiative capture

When a free proton and neutron form a deuteron,

$$p + n \to {}^2\mathrm{H} + \gamma, \tag{1.3}$$

the photon energy (minus a small recoil correction) equals B_d. The measured value is $2.224589 \pm 0.000002\,\mathrm{MeV}$, in excellent agreement with the mass-spectroscopic result.

1.2.4 Method 3: photodisintegration

The reverse process,

$$\gamma + {}^2\mathrm{H} \to p + n, \tag{1.4}$$

has a threshold equal to B_d (again corrected for recoil). Experiment gives $2.224 \pm 0.002\,\mathrm{MeV}$.

How weak is "weakly bound"?

Typical mid-mass nuclei have $B/A \sim 8\,\mathrm{MeV}$ (Lecture 4). The deuteron has $B/A \approx 1.11\,\mathrm{MeV}$. It sits far down on the rising part of the B/A curve: almost unbound. That single fact will control everything we calculate below.

For course work we adopt the rounded value used in the original lecture,

$$B_d \approx 2.225\,\mathrm{MeV}, \tag{1.5}$$

which is consistent with all three determinations at the level needed for the square-well analysis.

1.3 A simple model: the square well

1.3.1 Why a square well?

The true NN potential is complicated: a repulsive core at short distance, a strong attraction at $\sim 1\,\mathrm{fm}$, a pion-exchange tail at larger r, and spin- and tensor-dependent pieces (Lectures 7–9). To learn the *scale* of the force we represent the interaction by a three-dimensional square well in the relative coordinate $r = |\boldsymbol{r}_p - \boldsymbol{r}_n|$:

$$V(r) = \begin{cases} -V_0, & r < R, \\ 0, & r > R. \end{cases} \tag{1.6}$$

Here R is a measure of the range (roughly the "diameter" of the interaction region). The well depth V_0 is what we will fix from B_d.

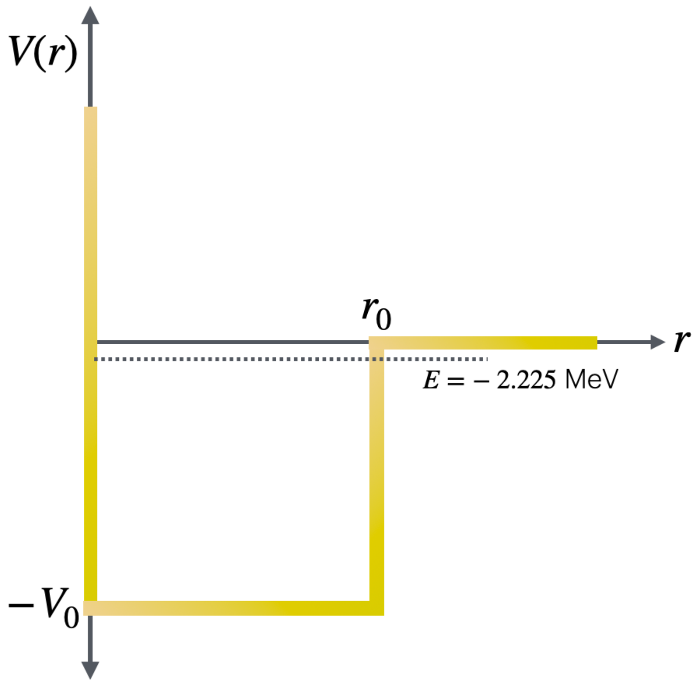

Figure 1.1: Square-well model of the NN potential (lecture figure). Inside $r < R$ the potential is $-V_0$; outside it vanishes. The deuteron energy $E = -B_d$ lies just below threshold.

1.3.2 Reduced mass and the radial equation

The two-body problem with equal masses $m_p \approx m_n \approx m_N$ reduces to a one-body problem with reduced mass

$$\mu = \frac{m_p m_n}{m_p + m_n} \approx \frac{m_N}{2}. \tag{1.7}$$

For a central potential we write the wave function as $\psi(\boldsymbol{r}) = Y_{\ell m}(\hat{r})\, u(r)/r$. The radial function $u(r)$ obeys

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \left[V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right]u = Eu. \tag{1.8}$$

Assumption (to be checked later): the ground state has $\ell = 0$, by analogy with the hydrogen $1s$ state. Then the centrifugal term vanishes and (1.8) looks exactly like a one-dimensional Schrödinger equation on the half-line $r > 0$.

1.3.3 Interior and exterior solutions

With $E = -B_d < 0$, define

$$k = \frac{1}{\hbar}\sqrt{2\mu(V_0 - B_d)} \qquad (r < R), \tag{1.9}$$

$$\gamma = \frac{1}{\hbar}\sqrt{2\mu B_d} \qquad (r > R). \tag{1.10}$$

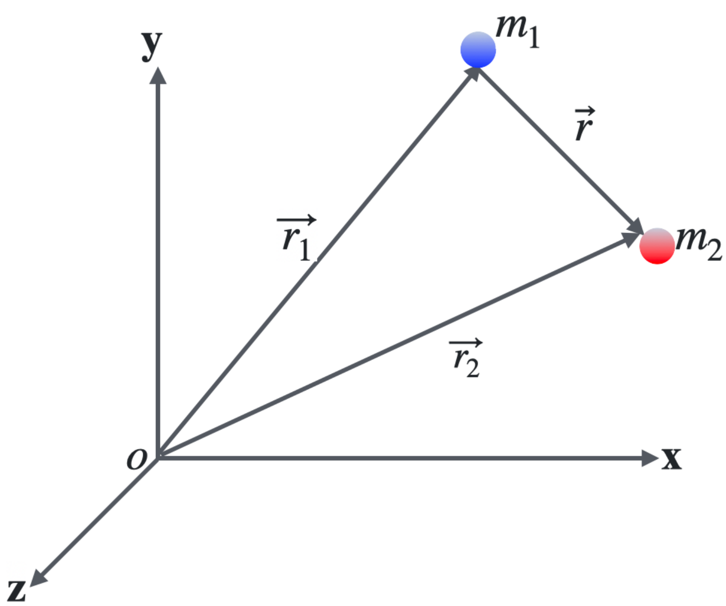

Figure 1.2: Relative coordinate of proton and neutron. The square well acts on this separation; the centre of mass moves freely.

(Equivalently $k = \sqrt{2\mu(V_0 + E)}/\hbar$ with $E = -B_d$. The lecture often writes $\hbar^2/2\mu \approx 41.3\,\text{MeV fm}^2$ for $\mu \approx m_N/2$.)

Inside the well the general solution is

$$u(r) = A\sin(kr) + B\cos(kr). \tag{1.11}$$

Boundary condition at the origin: ψ must remain finite, so $u(0) = 0$. That forces $B = 0$:

$$u(r) = A\sin(kr), \qquad r < R. \tag{1.12}$$

Outside the well,

$$u(r) = Ce^{-\gamma r} + De^{+\gamma r}. \tag{1.13}$$

Boundary condition at infinity: u must not explode, so $D = 0$:

$$u(r) = Ce^{-\gamma r}, \qquad r > R. \tag{1.14}$$

1.3.4 Matching: the transcendental condition

The physical content of the bound-state problem is fixed by the finite range of the force and by quantum mechanics. Outside the well there is no attraction, so a bound system ($E = -B_d < 0$) can exist only because the wave function tunnels under the continuum; the exterior solution must decay exponentially. Inside the well the nucleons feel a constant attraction $-V_0$, so the radial function oscillates. Matching the two regions at $r = R$ quantises the allowed (V_0, R) pairs that reproduce the measured binding energy [6, 13].

Write

$$V(r) = \begin{cases} -V_0 & r < R, \\ 0 & r > R, \end{cases} \qquad E = -B_d < 0.$$

For $r < R$ the radial Schrödinger equation becomes

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} - V_0 u = Eu \quad \Rightarrow \quad \frac{\mathrm{d}^2 u}{\mathrm{d}r^2} = -k^2 u,$$

with wave number

$$k = \frac{\sqrt{2\mu(V_0 - B_d)}}{\hbar} = \frac{\sqrt{2\mu(V_0 + E)}}{\hbar}, \tag{1.15}$$

where the second form uses $E = -B_d$ so that $V_0 - B_d = V_0 + E$ (not $V_0 + |E|$ written as a separate definition: since $E = -B_d$, one has $V_0 + E = V_0 - B_d$). Both expressions are identical; they are not two different physical definitions. The general interior solution is $u_{\mathrm{I}}(r) = A\sin(kr) + B\cos(kr)$. Because $\psi = u/r$ must remain finite at the origin, $u(0) = 0$, which forces $B = 0$:

$$u_{\mathrm{I}}(r) = A\sin(kr). \tag{1.16}$$

Physically this is the statement that there is no source of probability current at $r = 0$; the relative wave function vanishes at vanishing separation in the s-wave radial problem.

Outside the well, $V = 0$ and $E = -B_d$, so

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} = \gamma^2 u, \qquad \gamma = \frac{\sqrt{2\mu B_d}}{\hbar}.$$

The exterior solution is $u_{\mathrm{II}}(r) = Ce^{-\gamma r} + De^{+\gamma r}$. The growing exponential is discarded so that the state is normalisable:

$$u_{\mathrm{II}}(r) = Ce^{-\gamma r}. \tag{1.17}$$

The decay length $1/\gamma$ is large when B_d is small: weak binding means the nucleons spend much of their time *outside* the range of the force, which is why the deuteron rms radius exceeds the naive well radius.

Continuity of ψ and of $\mathrm{d}\psi/\mathrm{d}r$ at the edge of a finite square well requires continuity of u and u' at $r = R$:

$$A\sin(kR) = Ce^{-\gamma R}, \tag{1.18}$$

$$Ak\cos(kR) = -C\gamma e^{-\gamma R}. \tag{1.19}$$

Dividing these two equations eliminates the unknown amplitudes A and C and yields the matching condition

$$k\cot(kR) = -\gamma. \tag{1.20}$$

Equation (1.20) is transcendental: once R and B_d are fixed, it determines the depth V_0 that can support the observed bound state. That is the quantitative link between the measured binding energy and the strength of the residual force.

1.4 Numerical result: $V_0 \approx 35$–$36\,\mathrm{MeV}$

Electron scattering gives an rms charge radius of the deuteron of about 2.1 fm. That number is *not* the force range: because the binding is weak, the wave function extends well beyond the potential, so the matter/charge radius exceeds the interaction radius. Taking a force range $R \sim 2.0$–$2.1\,\mathrm{fm}$ as a first estimate, with $B_d = 2.224\,\mathrm{MeV}$ and $\hbar^2/(2\mu) \approx 41.5\,\mathrm{MeV\,fm}^2$, one solves (1.20) as follows.

Define the dimensionless quantities

$$\xi \equiv kR, \qquad \eta \equiv \gamma R = R\sqrt{\frac{2\mu B_d}{\hbar^2}}. \tag{1.21}$$

For $R = 2.1\,\mathrm{fm}$,

$$\eta = 2.1\sqrt{\frac{2.224}{41.5}} \approx 2.1 \times 0.231 \approx 0.486.$$

The matching condition $k\cot(kR) = -\gamma$ becomes $\xi \cot \xi = -\eta$, or

$$\xi \cot \xi = -0.486. \tag{1.22}$$

The relevant root in $(\pi/2, \pi)$ (the first bound state has $\xi > \pi/2$) is $\xi \approx 1.82$. Then

$$V_0 - B_d = \frac{\hbar^2 k^2}{2\mu} = \frac{\hbar^2}{2\mu}\frac{\xi^2}{R^2} = 41.5 \cdot \frac{(1.82)^2}{(2.1)^2} \approx 31.2\,\mathrm{MeV}, \tag{1.23}$$

$$V_0 = 31.2 + 2.224 \approx 33.4\,\mathrm{MeV}. \tag{1.24}$$

Varying R over 2.0–2.1 fm yields

$$V_0 \approx 33\text{–}35\,\mathrm{MeV}. \tag{1.25}$$

(A shorter assumed range requires a deeper well, and conversely.) Equation (1.20) constrains a curve in the (R, V_0) plane, not a unique pair: one observable (B_d) cannot fix two potential parameters. Either way, the residual NN attraction sampled by the deuteron is tens of MeV deep on a fermi scale, barely enough to bind [6, 13].

(The section title quotes ≈ 35–$36\,\mathrm{MeV}$ as a round classroom figure for $R \sim 2\,\mathrm{fm}$; the explicit root above gives ≈ 33–$35\,\mathrm{MeV}$ for the same range window.)

Key idea

Barely bound. The energy $E = -B_d$ lies just below the top of the well. If the force were slightly weaker, or the range slightly smaller, there would be no bound state at all. The wave function barely "turns over" inside the well so that it can match a decaying exponential outside [6].

Why this matters for the Sun and the early universe

Deuterium formation is the first step in the solar *pp* chain and in Big Bang nucleosynthesis. If no stable two-nucleon bound state existed, heavier nuclei would not form in the early universe and stellar hydrogen burning would not begin as it does. The weak binding of the deuteron is a threshold fact about the world we inhabit [6].

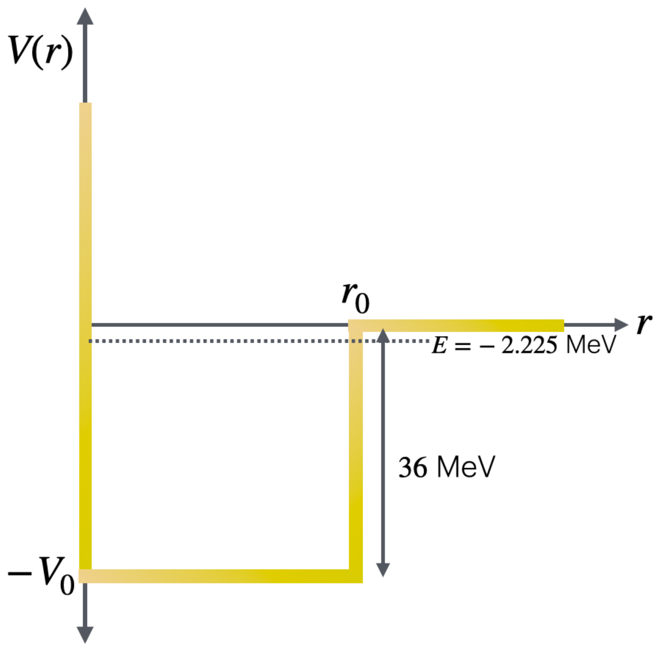

Figure 1.3: Lecture figure emphasising the depth V_0 needed to bind the deuteron. The bound level sits only $\sim 2.2\,\text{MeV}$ below the continuum, so most of the well depth is cancelled by kinetic energy of confinement.

Remark

Because the binding is weak, the probability of finding the nucleons *outside* the range R is large. The exponential tail extends far; low-energy NN scattering is correspondingly sensitive to the asymptotic normalisation. A large fraction of the time is spent outside the well [2].

1.5 Residual force: why nucleons attract at all

Before leaving the central model, place the force in a modern context. Nucleons are not elementary: $p = uud$, $n = udd$. The fundamental strong interaction (QCD) confines colour. Colour-neutral nucleons still interact at short distance through a residual force, much as neutral atoms form molecules through residual electromagnetic multipoles.

Yukawa's meson exchange (1935) explains the long-range tail: the force range is of order $\hbar/(m_\pi c) \sim 1.4\,\text{fm}$. The square well is a teaching caricature of that attraction plus short-range repulsion. Lectures 7–9 will show that even this picture is incomplete: the force depends on spin and contains a non-central (tensor) piece.

1.6 What we have learned, and what we have not

1. The deuteron binding energy is $B_d \approx 2.224\,\text{MeV}$, measured three independent ways.

2. There are no bound excited states: the system is barely bound.

3. A central square well with range $\sim 2\,\text{fm}$ requires depth $V_0 \sim 33$–$35\,\text{MeV}$ to bind; shorter range needs deeper V_0.

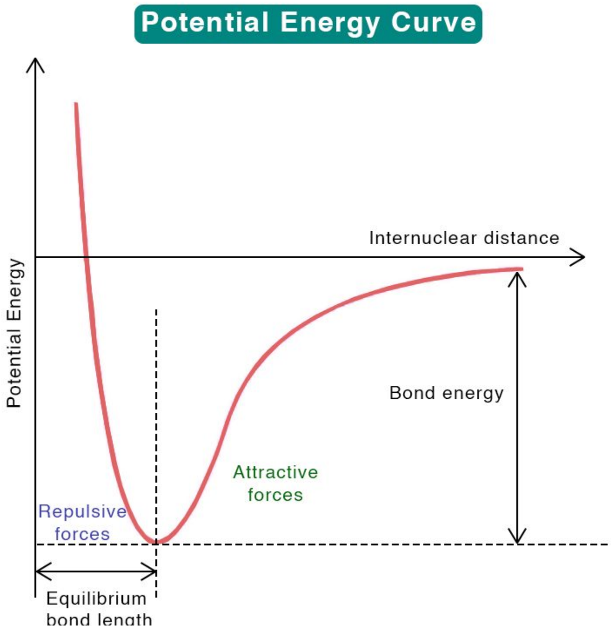

Figure 1.4: Molecular analogy: short-range repulsion, attractive well, finite range. Colour-neutral nucleons interact through residual colour forces on the fermi scale in a similar qualitative way.

4. That depth sets the scale of the NN attraction underlying the SEMF volume term a_V (Lecture 4).

5. We have *assumed* $\ell = 0$. Spin, magnetic moment, and quadrupole moment will test that assumption (Lectures 8–9).

Takeaway

In this lecture and the next.

- This lecture: binding energy and a central well $\Rightarrow$ force range and depth.
- Next lecture: I^π and $\mu_d \Rightarrow$ spin dependence and a few-percent D-wave.
- Lecture after: $Q_d \neq 0 \Rightarrow$ tensor (non-central) force.

Only after those three steps is the qualitative NN force complete for PH5001.

1.7 Deeper physics: weak binding and the asymptotic wave function

1.7.1 No bound excited states

Unlike the hydrogen atom, the deuteron has **no bound excited states**. Any excitation exceeds B_d and produces a free $p + n$ pair in the continuum. All structure information must therefore come from the ground-state multipoles (Lectures 8–9) and from scattering (phase shifts), not from a discrete spectrum [6, 2].

1.7.2 Wave function mostly outside the well

With $V_0 \sim 33$–$35\,\mathrm{MeV}$ and $R \sim 2\,\mathrm{fm}$, the matching condition $k\cot(kR) = -\gamma$ places the eigenenergy just below threshold. The exterior decay constant $\gamma = \sqrt{2\mu B_d}/\hbar$ is small ($\gamma^{-1} \approx 4.3\,\mathrm{fm}$), so the exponential tail extends far: the two nucleons spend a large fraction of the time *outside* the range of the force. That is why the rms charge radius ($\sim 2.1\,\mathrm{fm}$ from electron scattering) is not a direct measure of the force range, and why low-energy NN scattering is so sensitive to the asymptotic normalisation.

Range vs depth degeneracy

Fixing only B_d leaves a curve of allowed (R, V_0). Scattering lengths and effective ranges (Lecture 10) break that degeneracy by adding low-energy continuum information. A single bound-state energy never determines a unique potential.

1.7.3 Square well as a teaching potential

Realistic potentials (Yukawa one-pion tail, hard or soft core at short distance, tensor and spin–orbit terms) are more complex. The square well is retained because:

- it is analytically solvable for $L = 0$;
- it returns $V_0 \sim 33$–$40\,\mathrm{MeV}$ for $R \sim 2\,\mathrm{fm}$, the correct scale;
- it makes the near-threshold character of the bound state obvious.

Lectures 8–9 show why a pure central well is still incomplete: spin and quadrupole data demand a tensor component.

1.8 Further physics: only one bound NN state

There is only one bound $A = 2$ nucleus, the deuteron, and it has no bound excited states. Its binding energy is only $B_d \approx 2.225\,\mathrm{MeV}$, so any excitation puts the system into the $p + n$ continuum [14, 6, 13]. That single fact forces the entire multipole programme of Lectures 7–9: structure information must come from ground-state moments and from scattering, not from a discrete spectrum.

Modern NN potentials also include three-nucleon forces, which have no classical two-body analogue and are hard to fix from scattering alone [13, 4]. For PH5001 the two-body square well and the tensor force extracted from the deuteron remain the essential lessons.

At the QCD level, the residual nucleon–nucleon force is the long-distance face of a theory in which the strong coupling *grows* toward large distance (confinement) and becomes weak at short distance (asymptotic freedom) [15, 4]. Yukawa's pion sets the range ~ 1–$2\,\mathrm{fm}$ of the residual force used in this chapter.

1.8.1 Bound-state condition for a square well

For an s-wave square well of depth V_0 and range R,

$$V(r) = \begin{cases} -V_0 & r < R, \\ 0 & r > R, \end{cases} \tag{1.26}$$

the radial function $u(r) = r\psi(r)$ oscillates inside the well and decays as $e^{-\kappa r}$ outside, with

$$k = \frac{\sqrt{2\mu(V_0 + E)}}{\hbar}, \qquad \kappa = \frac{\sqrt{-2\mu E}}{\hbar}, \qquad E = -B_d < 0. \tag{1.27}$$

Matching u and u' at $r = R$ yields

$$k\cot(kR) = -\kappa. \tag{1.28}$$

The first bound state appears when the exterior decay constant vanishes, $\kappa \to 0^+$, so $\cot(kR) \to 0^-$ and

$$kR = \frac{\pi}{2} \qquad \text{(threshold)}. \tag{1.29}$$

At $E = 0$ one has $k = \sqrt{2\mu V_0}/\hbar$, hence at least one bound state requires

$$V_0 R^2 > \frac{\pi^2\hbar^2}{8\mu} \approx 109\,\mathrm{MeV\,fm}^2 \tag{1.30}$$

($\mu \approx m_N/2$). Fitting the deuteron gives approximately

$$V_0 R^2(s{=}1) \approx 140\,\mathrm{MeV\,fm}^2, \tag{1.31}$$

while low-energy np scattering in the singlet channel yields

$$V_0 R^2(s{=}0) \sim 90\text{–}95\,\mathrm{MeV\,fm}^2 \tag{1.32}$$

just *below* threshold. Typical phenomenological values are $V_0 \approx 47\,\mathrm{MeV}$, $R \approx 1.7\,\mathrm{fm}$ (triplet) and a shallower, wider singlet well [14, 6].

Because B_d is small, the exterior decay length $1/\kappa$ is long. Normalising the asymptotic wave $u(r) \sim Ae^{-\kappa r}$ for $r > R$, the probability outside the well is

$$P_> = \frac{\int_R^\infty A^2 e^{-2\kappa r}\,\mathrm{d}r}{\int_0^\infty u^2\,\mathrm{d}r} \sim \frac{A^2 e^{-2\kappa R}/(2\kappa)}{\text{total norm}}, \tag{1.33}$$

which is of order 50%–70% for the deuteron: most of the probability density sits *outside* the range of the force, and the rms radius exceeds R.

1.8.2 No nn or pp bound states and charge independence

Charge independence of the strong force, together with the Pauli principle, explains why only pn binds: the $s = 1$, $\ell = 0$ state is allowed for pn but forbidden for identical nn or pp; the $s = 0$ channel is unbound for all three pairs [14]. Mirror binding differences such as ^{3}H vs ^{3}He are then mostly Coulomb.

The naive reading "attractive in $s = 1$, repulsive in $s = 0$" is wrong: both channels are attractive; only the triplet is deep enough to bind [14, 6].

1.8.3 Yukawa potential and pion range

A more realistic long-range form is

$$V(r) = g\,\frac{\hbar c}{r}\,e^{-r/r_0}, \qquad r_0 = \frac{\hbar}{m_\pi c} \approx 1.4\,\text{fm}. \tag{1.34}$$

Yukawa predicted the pion mass from the force range [9, 14, 5]. In Born approximation the Fourier transform of the Yukawa potential is the propagator

$$M(q^2) \propto \frac{1}{q^2 + m_\pi^2 c^2}, \tag{1.35}$$

which reduces to a contact interaction when $m_\pi \to \infty$ (zero range). Short-range repulsion ("hard core") and multi-meson exchanges complete modern NN potentials [14, 15].

The same Yukawa logic, applied to the weak interaction with a heavy mediator, explains why the weak force looks feeble at low energy even though its underlying coupling is not tiny: $G_F \propto g^2/m_W^2$ [5]. For the strong residual force the light pion makes the range nuclear, not subnuclear.

1.8.4 Photodisintegration and the binding energy

The deuteron binding energy is measured to high precision from the threshold for

$$\gamma + {}^2\text{H} \to p + n, \tag{1.36}$$

and from the reverse radiative capture. The photon energy must supply at least $B_d \approx 2.224\,\text{MeV}$ (plus a small centre-of-mass correction). That B_d is only $\sim 0.1\%$ of the rest energy of the deuteron: the system is barely bound, which is why the wave function leaks so far outside the well [6, 14].

1.8.5 Why scattering is needed

With only one bound state, the deuteron samples the NN force mainly in the ${}^3S_1$–${}^3D_1$ channel at a fixed average separation. Nucleon–nucleon scattering (especially on hydrogen targets, to avoid multiple scattering inside a heavy nucleus) maps other partial waves, energies, and spin orientations. Phase shifts then constrain the full potential used in few-body and many-body calculations [6, 4].

1.8.6 Triplet vs singlet wells in numbers

Phenomenological square-well fits that also use low-energy scattering give distinct wells in the two spin channels [14]:

Channel	V_0 (MeV)	R (fm)	V_0R^2 (MeV fm^2)
Triplet $s = 1$ (bound)	46.7	1.73	≈ 140
Singlet $s = 0$ (unbound)	12.5	2.79	≈ 93.5
Threshold for one bound state			109

Both channels are attractive; only the triplet clears the quantum threshold $V_0R^2 > 109$ MeV fm^2. That is the quantitative content of "spin-dependent nuclear force" at the square-well level.

1.9 Problems with solutions

Problem 7.1 — Binding energy of the deuteron

Atomic masses: $m(^1\text{H}) = 1.007825\,\text{u}$, $m(n) = 1.008665\,\text{u}$, $m(^2\text{H}) = 2.014102\,\text{u}$. With $c^2 = 931.5\,\text{MeV/u}$, compute B_d and B_d/A.

Solution 7.1

$$B_d = [m(^1\text{H})+m(n)-m(^2\text{H})]\,c^2 = (1.007825+1.008665-2.014102)\times 931.5 = 0.002388\times 931.5 \approx 2.224\,\text{MeV}.$$

$$B_d/A \approx 1.11\,\text{MeV},$$

much less than the mid-mass ~ 8 MeV per nucleon: the deuteron is barely bound [6, 14].

Problem 7.2 — Uncertainty estimate of V_0

For a square well of range $R = 2.1$ fm and reduced mass $\mu = m_N/2$, use $E_{\text{kin}} \sim 3\hbar^2/(2\mu R^2)$ with $\hbar^2/m_N \approx 41.5$ MeV fm^2 and $B_d = 2.2$ MeV to estimate V_0.

Solution 7.2

Since $\hbar^2/(2\mu) = \hbar^2/m_N$,

$$E_{\text{kin}} \sim \frac{3\times 41.5}{(2.1)^2} \approx 28\,\text{MeV}, \qquad V_0 \sim E_{\text{kin}} + B_d \sim 30\,\text{MeV}.$$

Same tens-of-MeV scale as the matching solution $V_0 \approx 35$ MeV [6].

Problem 7.3 — Bound-state condition V_0R^2 (Basdevant)

For an s-wave square well, at least one bound state requires $V_0R^2 > \pi^2\hbar^2/(8\mu) \approx 109$ MeV fm^2. (a) With $R = 1.73$ fm and $V_0 = 46.7$ MeV (triplet fit), compute V_0R^2 and compare with 109 MeV fm^2. (b) The singlet channel has $V_0R^2 \sim 93.5$ MeV fm^2. What does that imply?

Solution 7.3

(a)

$$V_0R^2 = 46.7 \times (1.73)^2 \approx 46.7 \times 2.99 \approx 140\,\text{MeV fm}^2 > 109\,\text{MeV fm}^2.$$

The triplet well is above threshold and binds the deuteron ($V_0R^2 \approx 140\,\text{MeV fm}^2$ reproduces $B_d = 2.225\,\text{MeV}$) [14].

(b) $93.5 < 109$: the singlet well is just *below* threshold. There is no bound $^1S_0$ pn state; low-energy singlet scattering shows a virtual state [14, 6].

Problem 7.4 — Exterior decay length

With $B_d = 2.224\,\text{MeV}$ and $\hbar^2/m_N = 41.5\,\text{MeV fm}^2$, compute $\gamma = \sqrt{2\mu B_d}/\hbar = \sqrt{m_N B_d}/\hbar$ and the decay length $1/\gamma$. Compare with a typical well range $R \sim 2\,\text{fm}$.

Solution 7.4

$$\gamma = \sqrt{\frac{m_N B_d}{\hbar^2}} = \sqrt{\frac{B_d}{\hbar^2/m_N}} = \sqrt{\frac{2.224}{41.5}} \approx \sqrt{0.0536} \approx 0.231\,\text{fm}^{-1}.$$

$$\frac{1}{\gamma} \approx 4.3\,\text{fm} > R \sim 2\,\text{fm}.$$

Most of the probability density lies outside the range of the force: the deuteron is a loosely bound, extended system [6, 14].

Problem 7.5 — Thermal deuterons (Petrera 1.1.4)

Estimate the temperature at which two deuterons can approach to $r \sim 1\,\text{fm}$ against Coulomb repulsion. Use $k_B = 8.6 \times 10^{-5}\,\text{eV/K}$ and $\alpha\,\hbar c \approx 1.44\,\text{MeV fm}$.

Solution 7.5

In the rest frame of one deuteron the other approaches with relative velocity $2v$, so

$$4k_BT \sim \frac{\alpha\,\hbar c}{r} \approx \frac{1.44}{1} = 1.44\,\text{MeV}.$$

$$T \sim \frac{1.44 \times 10^6\,\text{eV}}{4 \times 8.6 \times 10^{-5}\,\text{eV/K}} \sim 4 \times 10^9\,\text{K}.$$

Nuclear contact among charged light nuclei requires stellar temperatures [3].

Problem 7.6 — Photodisintegration threshold

The $\gamma + {}^2\text{H} \to p + n$ threshold is essentially B_d plus a small centre-of-mass correction. If a photon of energy $E_\gamma = 2.50\,\text{MeV}$ breaks a deuteron at rest, what is the total kinetic energy of the $p + n$ system in the lab (neglect recoil of the compound system before breakup, to leading order)?

Solution 7.6

Energy conservation:

$$E_\gamma = B_d + T_p + T_n \quad \Rightarrow \quad T_p + T_n = 2.50 - 2.224 = 0.276\,\mathrm{MeV}.$$

The excess photon energy above threshold appears as kinetic energy of the two nucleons [6].

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