Chapter 1

Theories of Nuclear Forces

"Yukawa noticed that the range of nuclear forces $r_0 \simeq 1.4\,\mathrm{fm}$ *corresponds to a particle of mass* $m_\pi c^2 \simeq 140\,\mathrm{MeV}$*."*

paraphrasing H. Yukawa (1935)

Lectures 7–10 constrain the residual NN force from two sides: the deuteron multipoles and low-energy phase shifts. Both approaches are phenomenological: they fix a potential (or its phase shifts) to data. This lecture asks a deeper question: *what carries the force?* In modern quantum field theory, forces between particles arise from the exchange of virtual quanta. For electromagnetism the quantum is the massless photon and the potential is Coulomb. For the residual strong force the lightest quantum is the massive pion and the potential is Yukawa [9, 14, 5, 15, 2].

1.1 Three layers of description: from Coulomb to QED

Electromagnetism shows how the same interaction can be told at increasing depth.

Action at a distance. Two charges q_1 and q_2 exert equal and opposite forces

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \tag{1.1}$$

instantaneously across empty space. The formula works at low speeds, but instantaneous action conflicts with relativity: no influence can travel faster than light.

Classical field. Charge q_1 does not "reach out" to q_2. It produces an electric field filling space; q_2 feels a force because it sits in that field. The field is the intermediary. Changes in the field propagate at finite speed c.

Quantum field (QED). The electromagnetic field is quantized. Energy and momentum are exchanged in discrete packets (photons). The Coulomb force is the low-energy limit of exchanging *virtual* photons: off-shell quanta that mediate the interaction without appearing as free radiation. Virtual photons share the quantum numbers of real photons (massless, chargeless, spin 1); only their four-momentum differs from the on-shell condition $q^2 = 0$ [5, 15].

The same ladder applies to nucleons N_1 and N_2. Classically one might imagine a potential $V(r)$. At the quantum level the interaction is the exchange of virtual mesons. Unlike the photon, those mesons may carry mass and electric charge, and that changes the range and the selection rules of the force.

1.2 Range of a force from the uncertainty principle

A useful *heuristic* estimates the range of a force mediated by a massive quantum. Creating a virtual particle of rest energy mc^2 requires an energy fluctuation of order $\Delta E \sim mc^2$. The uncertainty principle then suggests a lifetime

$$\Delta t \lesssim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{mc^2}. \tag{1.2}$$

Even if the quantum travels near the speed of light, the farthest it can go before being reabsorbed is of order

$$R \lesssim c\,\Delta t \sim \frac{\hbar c}{mc^2} = \frac{\hbar}{mc}. \tag{1.3}$$

This Compton wavelength of the mediator sets the force range. Two limits are immediate:

- **Massless mediator** ($m = 0$): $R \to \infty$. The force is long-ranged (Coulomb, gravity in the Newtonian limit).
- **Massive mediator**: R is finite. The force is short-ranged; beyond a few Compton wavelengths the exchange is exponentially suppressed.

This argument is pedagogical, not a derivation of field theory: it does not claim that energy conservation is violated. The rigorous statement is that a massive propagator $1/(q^2 + m^2c^2)$ Fourier-transforms to an exponentially screened potential, derived next from the Klein–Gordon equation [9, 14, 5, 2].

That single scale explains why nuclear forces die within a few fm while electrostatics still acts across atomic distances.

1.2.1 Predicting the pion mass from the nuclear range

Electron scattering and low-energy NN data fix the range of the residual strong force at $R \sim 1$–$2\,\mathrm{fm}$. Taking $R \approx 1.4$–$2\,\mathrm{fm}$ and $\hbar c \approx 197\,\mathrm{MeV\,fm}$,

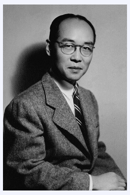
$$mc^2 \sim \frac{\hbar c}{R} \approx \frac{197}{1.4\text{–}2} \approx 100\text{–}140\,\mathrm{MeV}. \tag{1.4}$$

Yukawa made this prediction in 1935, years before the pion was discovered. The observed mesons have

$$m_{\pi^\pm}c^2 \approx 139.6\,\mathrm{MeV}, \qquad m_{\pi^0}c^2 \approx 135\,\mathrm{MeV},$$

exactly the scale required by the nuclear force range [9, 14, 2].

Historical note: Hideki Yukawa (1907–1981)



Hideki Yukawa, working in Osaka in 1934–1935, proposed that the short-range nuclear force is mediated by the exchange of a massive boson. From the empirical range $r_0 \sim 1$–$2\,\mathrm{fm}$ he estimated a mediator mass of order $100\,\mathrm{MeV}/c^2$. The particle, the pion, was later found with $m_\pi c^2 \approx 140\,\mathrm{MeV}$, and the associated potential falls exponentially with range fixed by m_π. Yukawa received the 1949 Nobel Prize in Physics, the first Nobel awarded to a Japanese scientist. His insight is the microscopic origin of the residual NN force developed in this lecture [9, 14, 2].

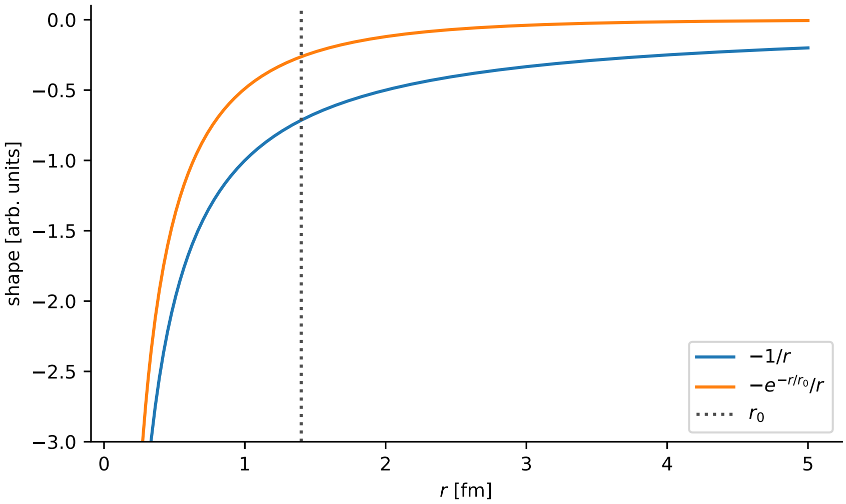


Figure 1.1: Coulomb ($-1/r$, massless exchange) versus Yukawa ($-e^{-r/r_0}/r$, massive exchange). The exponential kills the force beyond $r_0 \sim \hbar/(m_\pi c)$.

Key idea

Finite nuclear range $\Leftrightarrow$ massive exchange quantum. The pion mass and the fermi-scale range of the residual NN force are two faces of $R \sim \hbar/(m_\pi c)$.

1.3 From the Klein–Gordon equation to the Yukawa potential

The form of the potential follows from the relativistic wave equation for a spinless field of mass m. We use $\boldsymbol{r}$ in fm, $\boldsymbol{q}$ and $\mu = mc/\hbar$ in fm^{-1}, and energies in MeV; g is dimensionless, so $\hbar c$ is displayed explicitly. The free Klein–Gordon equation is

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \mu^2\psi = 0, \qquad \mu = \frac{mc}{\hbar}. \tag{1.5}$$

For $m = 0$ this reduces to the wave equation for electromagnetic potentials. A static point source for the massless case yields Coulomb's law. To keep source strength, test-particle coupling, and sign distinct, let a positive source g generate a scalar field ϕ through

$$(\nabla^2 - \mu^2)\phi(r) = -g\hbar c\,\delta^3(\boldsymbol{r}), \qquad V(r) = -g\phi(r). \tag{1.6}$$

The last relation chooses the attractive scalar channel; changing the coupling or operator channel can change the sign of the physical potential. For $r \neq 0$ the radial equation is

$$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(r\phi) - \mu^2\phi = 0,$$

so $r\phi = Ae^{-\mu r} + Be^{+\mu r}$. Boundedness at infinity forces $B = 0$, hence $\phi(r) = Ae^{-\mu r}/r$. To fix A, integrate (1.6) over a ball of radius $\varepsilon \to 0$:

$$\int_{r<\varepsilon}(\nabla^2 - \mu^2)\phi\,\mathrm{d}^3r = \oint_{\partial B}\nabla\phi\cdot\mathrm{d}\boldsymbol{S} - \mu^2\int\phi\,\mathrm{d}^3r = -g\hbar c. \tag{1.7}$$

The volume integral of $\phi \sim 1/r$ vanishes as $\varepsilon \to 0$, while $\nabla\phi \sim -A\hat{r}/r^2$ gives $\oint\nabla\phi\cdot\mathrm{d}\boldsymbol{S} = -4\pi A$. Therefore $A = g\hbar c/(4\pi)$, and the chosen test coupling gives

$$V(r) = -\frac{g^2}{4\pi}\frac{\hbar c}{r}\,e^{-\mu r} = -\frac{g^2}{4\pi}\frac{\hbar c}{r}\,e^{-r/r_0}, \qquad r_0 = \frac{\hbar}{mc} \approx 1.4\,\mathrm{fm} \quad (\text{for } m = m_\pi). \tag{1.8}$$

This is a pedagogical scalar Yukawa well. Physical one-pion exchange has spin–isospin operators, not a universal central attraction.

Fourier-transform route. The same potential is the Fourier transform of the massive propagator. In the static Born approximation,

$$\tilde{V}(\boldsymbol{q}) = -\frac{g^2\hbar c}{q^2 + \mu^2}, \tag{1.9}$$

and

$$\begin{aligned} V(r) &= \int\frac{\mathrm{d}^3q}{(2\pi)^3}\,\tilde{V}(\boldsymbol{q})\,e^{i\boldsymbol{q}\cdot\boldsymbol{r}} = -\frac{g^2\hbar c}{2\pi^2}\int_0^\infty\frac{q^2\,\mathrm{d}q}{q^2+\mu^2}\frac{\sin(qr)}{qr} \\ &= -\frac{g^2\hbar c}{4\pi}\frac{e^{-\mu r}}{r}, \end{aligned} \tag{1.10}$$

recovering (1.20). In the limit $m \to 0$ ($\mu \to 0$) one recovers the Coulomb shape $1/r$; for finite m the exponential cuts off the force beyond r_0 [14, 5].

In quantum field theory the same potential is the Fourier transform of the propagator $1/(q^2 + m^2c^2)$ of the exchanged meson in the static Born approximation [14, 5].

1.4 Pions as mediators of the residual strong force

Real pions are copiously produced in accelerators and in cosmic-ray showers in the atmosphere. Virtual pions of the same mass and quantum numbers can be exchanged between nucleons; the associated range is their Compton wavelength $\hbar/(m_\pi c) \sim 1.4\,\mathrm{fm}$.

It is crucial to distinguish the **fundamental** strong force from the **residual** force between nucleons. Inside a nucleon, quarks exchange gluons and the interaction is extremely strong and confining. A proton or neutron as a whole is a colour singlet ("white"). When two colour-neutral nucleons approach within a fermi, residual colour forces remain, analogous to van der Waals forces between neutral atoms. Pion exchange is the longest-range piece of that residual interaction

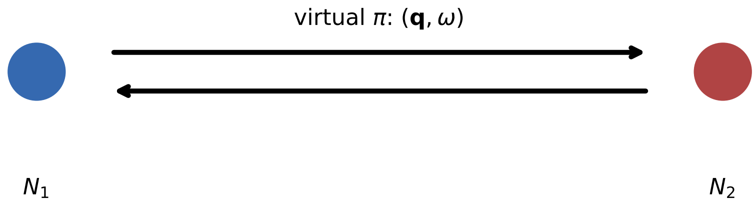


Figure 1.2: Schematic meson exchange between two nucleons. The virtual pion mediates a short-range residual force of Compton wavelength $\hbar/(m_\pi c)$.

[15, 4, 14].

1.4.1 Charge exchange: *pp* versus *np*

Neutral pion exchange preserves nucleon identity:

$$p \to p + \pi^0, \qquad n \to n + \pi^0. \tag{1.11}$$

Charged pion exchange converts neutron $\leftrightarrow$ proton:

$$n \to p + \pi^-, \qquad p + \pi^- \to n, \tag{1.12}$$

$$p \to n + \pi^+, \qquad n + \pi^+ \to p. \tag{1.13}$$

Consequently:

- In *pp* (or *nn*) scattering, if both nucleons keep their charges, only π^0 exchange is allowed at the one-pion level.
- In *np* scattering, π^0, π^+, and π^- can all contribute.

Because $m_{\pi^\pm}$ and m_{π^0} differ by only a few MeV, the *pp* and *np* forces are nearly equal once the Coulomb repulsion between protons is removed. That near-equality is the charge independence of the residual strong force, already used when arguing that only the *pn* triplet binds (Lecture 8) [14, 6].

1.5 One-pion exchange, tensor force, and the short-range core

Single-pion exchange (OPE) produces not only the radial Yukawa factor (1.20) but also spin-dependent and tensor structures. Put $\mu_\pi = m_\pi c/\hbar$, so both q and μ_π have dimensions of inverse length. In one common static pseudovector convention,

$$\widetilde{V}_\pi(\boldsymbol{q}) = -\frac{f_{\pi NN}^2}{\mu_\pi^2}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{q^2 + \mu_\pi^2}\,\hbar c, \tag{1.14}$$

where $f_{\pi NN}$ is dimensionless. Since multiplication by $q_i q_j$ becomes differentiation of $Y(r) = e^{-\mu_\pi r}/(4\pi r)$, decompose the two derivatives using

$$S_{12}(\hat{r}) = 3(\boldsymbol{\sigma}_1 \cdot \hat{r})(\boldsymbol{\sigma}_2 \cdot \hat{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2. \tag{1.15}$$

The coordinate-space result can then be written

$$V_\pi(\boldsymbol{r}) = C_\pi(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)W_S(r) + S_{12}(\hat{r})W_T(r)\right], \tag{1.16}$$

where the overall C_π depends on the coupling convention, while the distributional radial factors are

$$W_S(r) = \frac{\mu_\pi^2}{3}\frac{e^{-\mu_\pi r}}{r} - \frac{4\pi}{3}\delta^3(\boldsymbol{r}), \tag{1.17}$$

$$W_T(r) = \frac{\mu_\pi^2}{3}\frac{e^{-\mu_\pi r}}{r}\left(1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2}\right). \tag{1.18}$$

The delta-function is the zero-range part generated by differentiating $Y(r)$. A point-nucleon OPE formula is not trustworthy at $r = 0$: phenomenological potentials regulate this term, while effective field theory absorbs its short-distance content into contact operators. The finite-range spin–spin and tensor tails are the model-independent lesson. The potential piece $V_T(r)S_{12}$ is attractive in the deuteron channel and is the long-range origin of the S–D mixing, the quadrupole moment $Q_d \neq 0$, and the small correction to μ_d discussed in Lecture 9 [14, 6]. In multi-nucleon systems the tensor force largely averages out; in the two-body problem it is essential for binding.

A pure attractive Yukawa well is not enough for nuclear saturation. Phenomenological and meson-exchange potentials add a strong **repulsive core** at $r \lesssim 0.5\,\text{fm}$, often modelled as another Yukawa with a heavier mass (or as ω and multi-pion exchange). Heavier mediators have shorter Compton wavelengths, so they act only at short distance, exactly where the hard core is needed to keep nucleons from collapsing into a single blob of pion-range size [14, 4].

A realistic NN potential therefore has a hierarchy of ranges [14]:

Distance	Dominant physics	Mediators / language
$r \gtrsim 1.4\,\text{fm}$	long-range attraction, tensor	π (OPEP)
$r \sim 0.7$–$1.4\,\text{fm}$	intermediate attraction	multi-π, ρ
$r \lesssim 0.5\,\text{fm}$	short-range repulsion	ω, quark Pauli / contact terms in EFT

Modern chiral effective field theory organises the same hierarchy as an expansion in soft momenta: one-pion exchange at long range, multi-pion exchange at intermediate range, and short-range contact operators fitted to phase shifts. For this course it is enough that OPEP explains the long-range tensor force while shorter-range physics is constrained by scattering data (Lecture 10), not invented from the deuteron alone.

1.5.1 Why two-nucleon forces are not the whole Hamiltonian

Integrating out unresolved pion and nucleon excitations also generates irreducible three-nucleon (3N) operators. The classic mechanism is two-pion exchange in which one nucleon is virtually excited while interacting with two neighbours. Modern chiral EFT therefore contains consistent NN, 3N, and higher-body terms in a controlled hierarchy. 3N forces are smaller than the leading NN interaction but are needed quantitatively for few-body binding, nuclear saturation, shell evolution, and neutron-rich matter. They must not be mimicked by simply retuning the long-range OPE tail [6, 14].

1.5.2 Phenomenology and mechanism

Historically, NN potentials were first fitted to deuteron properties and scattering phase shifts with adjustable strengths and ranges (phenomenology). Yukawa's insight and modern one-boson-exchange models derive the long-range form from a field-theoretic mechanism. The two approaches meet in practice: phase shifts (Lecture 10) and the deuteron multipoles (Lectures 7–9) remain the data that any mechanism-based potential must reproduce [6, 14].

Observable (L7–10)	Inferred operator	Mechanism (this lecture)
B_d, no excited states	depth/range of attraction	residual force scale
$I^\pi = 1^+$, triplet binds	spin dependence	$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$Q_d \neq 0$, μ_d deficit	tensor force S_{12}	OPEP tensor
$a_t > 0$, $a_s < 0$, $\delta_\ell(E)$	channel-dependent V	OPE + shorter range

Weak interactions provide a contrast. Exchange of the heavy Z^0 ($m_Z c^2 \approx 91\,\text{GeV}$) yields a Yukawa range $\hbar c/m_Z c^2 \sim 2\times10^{-3}$ fm and an effective $V_0 R^2$ five orders of magnitude smaller than the strong residual force, negligible for nuclear binding though essential for neutrino scattering [14].

Key idea

Lectures 7–11 in one line. The deuteron and phase shifts map the residual NN potential; pion exchange explains why that potential is short-ranged, spin- and tensor-dependent, and charge-independent at long distance.

1.6 Deeper physics: Yukawa range and the force hierarchy

1.6.1 Compton wavelength as force range

The lecture estimates the range of an interaction mediated by a particle of mass m as $R \sim \hbar/(mc)$ (Equation (1.3)). For the pion,

$$r_\pi = \frac{\hbar}{m_\pi c} \approx \frac{197\,\text{MeV fm}}{140\,\text{MeV}} \approx 1.4\,\text{fm}, \tag{1.19}$$

matching the NN force range inferred from phase shifts and from the deuteron rms radius (Lecture 7). Yukawa's 1935 insight was to invert this logic: a force of nuclear range implies an exchanged quantum of pion mass [9, 14].

The Yukawa potential

$$V_Y(r) = g^2\,\frac{\hbar c}{r}\,e^{-r/r_\pi} \tag{1.20}$$

Fourier-transforms to a propagator $\propto 1/(q^2 + m_\pi^2 c^2)$. At momenta $q \ll m_\pi c$ the potential looks Coulomb-like ($\propto 1/r$); at $qr_\pi \gg 1$ it is exponentially suppressed. Nuclear saturation (Lecture 2) requires that attraction turn to repulsion at shorter distance, so one-pion exchange is necessary but not sufficient for a realistic NN potential.

1.6.2 Three rungs on the force ladder

Key idea

Force hierarchy (order of magnitude).

1. **QCD binding** inside a nucleon: $E \sim m_\pi c^2 \sim 140\,\text{MeV}$ confinement scale.
2. **Residual NN force:** $B_d \sim 2\,\text{MeV}$, range $\sim$ 1–2 fm.
3. **Electromagnetic:** Coulomb between protons, $\alpha\hbar c/r$, long ranged but $\alpha \approx 1/137$.

The residual force is $\sim 10^{-2}$ of the strong scale because nucleons are colour singlets: gluon exchange is suppressed, leaving pion exchange as the leading colour-neutral interaction [15, 4].

Weak force range for comparison

Replacing m_π by $m_W \approx 80\,\text{GeV}$ in (1.19) gives a range $\sim 10^{-3}\,\text{fm}$, explaining why weak decays appear as point interactions at nuclear scales while pion exchange still shapes the NN potential [5].

Remark

Beyond one pion. Heavy mesons (ρ, ω, σ-like states) contribute at $r \lesssim 1\,\text{fm}$ and generate the repulsive core seen in phase shift analyses (Lecture 10). Chiral EFT organises these contributions by momentum scale, linking the long-range pion physics of this lecture to the three-nucleon forces needed for $A > 2$ binding (Lecture 12 further material).

1.6.3 Virtual pions and the residual coupling

The pions exchanged between nucleons are *virtual*: their four-momentum satisfies $q^2 \neq m_\pi^2 c^2$, so the exchange duration is $\Delta t \sim \hbar/(m_\pi c^2)$ as in Equation (1.2). The coupling $g_{\pi NN}$ is larger than α, but the Yukawa factor e^{-r/r_π} suppresses the force at distance. That combination explains why the residual force binds nuclei ($B/A \sim 8\,\text{MeV}$) yet remains feeble compared with quark confinement inside each nucleon [15, 14, 5].

1.6.4 Charge independence and pion exchange

One-pion exchange respects approximate charge independence: π^0 and $\pi^\pm$ exchanges relate pp, nn, and pn channels. Coulomb breaks the symmetry for pp, which is why only the np deuteron binds (Lecture 7). The range estimate (1.19) is tied to the same physics that organises triplet and singlet scattering lengths in Lecture 10 [14, 9].

1.6.5 Electromagnetic vs strong hierarchy at 1 fm

At $r = 1\,\mathrm{fm}$, Coulomb repulsion between two protons is $\sim e^2/(4\pi\varepsilon_0 r) \approx 1.4\,\mathrm{MeV}$, comparable to a fraction of the NN well depth but far below the strong scale inside each nucleon. Pion exchange provides $\sim 10\,\mathrm{MeV}$ attraction at the same distance, overwhelming Coulomb for *np* pairs. The hierarchy table in this lecture is therefore quantitative: charge independence works for *np* and *nn* scattering because the residual strong force dominates Coulomb at nuclear separation [14, 5, 15].

Physics-depth close

If you remember only one equation from Lecture 11, let it be $R \approx \hbar/(m_\pi c)$. Everything else in modern force models is an organised correction around that long-distance scale.

1.7 Further physics: residual force and colour neutrality

Nucleons are colour singlets. The fundamental strong interaction binds quarks via gluon exchange and confines colour. The force between two nucleons is a residual, colour-van-der-Waals effect: short-ranged, saturating, and much weaker than the force inside a single nucleon [15, 4, 14]. Pion exchange is the longest piece of that residual force; heavier mesons and quark substructure dominate only at sub-fermi distances where the repulsive core appears.

1.7.1 Yukawa range: a numerical derivation

Inserting $m_\pi c^2 = 140\,\mathrm{MeV}$ and $\hbar c = 197\,\mathrm{MeV\,fm}$,

$$r_\pi = \frac{\hbar}{m_\pi c} = \frac{197}{140}\,\mathrm{fm} \approx 1.41\,\mathrm{fm}. \tag{1.21}$$

The same estimate applied to the ρ meson ($m_\rho c^2 \approx 770\,\mathrm{MeV}$) gives $r \sim 0.26\,\mathrm{fm}$: heavier mediators are shorter ranged and dominate the inner part of the NN potential where the repulsive core appears in phase-shift analyses (Lecture 10). The pion sets the *outer* shape; multiple exchanges build the inner repulsion [14, 9].

1.7.2 Force hierarchy in numbers

Compare energy scales at $r \sim 1\,\mathrm{fm}$:

Interaction	Scale at 1 fm	Range
Strong (QCD)	$\sim 100\,\mathrm{MeV}$ binding	confinement
Residual NN	$\sim 10\,\mathrm{MeV}$ (well depth)	$\sim 1.4\,\mathrm{fm}$
Coulomb (two protons)	$\sim 1.4\,\mathrm{MeV}$	long
Weak	$\ll 1\,\mathrm{keV}$	$\sim 10^{-3}\,\mathrm{fm}$

The residual force is weak because colour-singlet nucleons exchange colour-neutral mesons (Lecture 11 main text). Coulomb is long ranged but multiply charged by $\alpha \approx 1/137$. The deuteron binding energy $B_d \sim 2\,\mathrm{MeV}$ (Lecture 7) sits naturally on the residual row, not the QCD row [15, 5].

Pion mass from range (Yukawa 1935)

If nuclear forces vanish beyond $R = 2\,\mathrm{fm}$, then $mc^2 \sim \hbar c/R \approx 100\,\mathrm{MeV}$, bracketing the pion mass before it was discovered experimentally. The factor-of-two agreement is fortuitous (the force does not cut off sharply), but the *order* of magnitude established meson exchange as the carrier of the residual strong force [9, 14].

1.7.3 Bridge to deuteron and saturation

One-pion exchange generates the tensor force of Lecture 9 and contributes to the spin–orbit splittings of Lecture 12. Without short-range repulsion (from heavier exchanges), however, nuclear matter would collapse: the saturation density of Lecture 2 requires a balance between attraction at $r \sim 1$–$2\,\mathrm{fm}$ and repulsion at $r \lesssim 1\,\mathrm{fm}$. Three-nucleon forces (Lecture 11 research frontier) provide additional repulsion needed for correct $A > 2$ binding [41, 42].

1.7.4 Two-pion vs one-pion exchange

At $r \lesssim 1\,\mathrm{fm}$, two-pion exchange and heavier meson (ρ, ω) diagrams contribute comparable strength to one-pion exchange. Their short range produces the repulsive core required for saturation (Lecture 2) without altering the long-range a_t fixed by one-pion physics. Chiral EFT orders these contributions by momentum scale, so the Yukawa range of this lecture is the *leading* term in a controlled expansion rather than the complete potential [14, 4].

1.8 Deeper reading: range, hierarchy, and three-nucleon forces

Yukawa's relation $R \sim \hbar/(mc)$ with $m = m_\pi$ yields $\sim 1.4\,\mathrm{fm}$, matching the internucleon spacing of Lecture 2. Heavier mesons generate shorter ranges; contact terms absorb unresolved short-distance physics in an EFT organisation. Three-nucleon forces appear at a definite chiral order and are required for saturation: fitting only NN data typically misplaces binding beyond $A \sim 3$.

Students should estimate $m_\pi c^2 = 140\,\mathrm{MeV}$ and $\hbar c = 197\,\mathrm{MeV\,fm}$ once by hand to own the range formula. Similarity renormalisation group evolutions later soften the interaction for many-body methods (Lecture 12), but they induce many-body operators that must be tracked. The lecture arc is: deuteron phenomenology (Lectures 7–9), scattering constraints (Lecture 10), and meson-exchange organisation (Lecture 11) before mean-field shells.

Power counting intuition

Leading one-pion exchange is fixed by chiral symmetry and the pion-nucleon coupling; contacts absorb short-range physics order by order. Truncation errors can be estimated by comparing successive orders rather than by a single χ^2. That culture of uncertainty is the modern replacement for quoting one preferred hard-core potential.

Softening and induced forces

SRG evolution decouples high-momentum matrix elements, improving many-body convergence, but induces three- and four-body operators. Discarding induced many-body pieces can reintroduce saturation errors. Lecture 11's range argument still sets the physical long-distance scale those softened interactions must preserve.

Mass–range table

Compute $\hbar/(mc)$ for $m = m_\pi, m_\rho, m_N$. The progression $1.4\,\text{fm} \to 0.3\,\text{fm} \to 0.2\,\text{fm}$ organises long-range tensor physics, short-range repulsion, and unresolved contact structure. Chiral EFT keeps the pion explicit and buries the rest in contacts ordered by momentum. That is a bookkeeping revolution relative to listing every meson by name, but Yukawa's range formula remains the intuition check.

For three-nucleon forces, compare published binding energies of ^{3}H and ^{4}He with and without $3N$ terms in a given Hamiltonian family (literature values suffice). The shift is mega-electronvolt-scale, comparable to the entire deuteron binding: bulk saturation cannot ignore it. Lecture 2's Fermi-gas estimate assumed an effective interaction; Lecture 11 finally says where that interaction comes from.

Meson exchange organises the residual force by range. Yukawa's formula remains the intuition, while chiral EFT supplies the modern bookkeeping for contacts and three-nucleon forces.

In practice, rework one numerical estimate from the lecture with a different parameter set and compare the shift to the quoted experimental uncertainty. If the shift exceeds the uncertainty, the estimate is teaching a scale, not delivering a fit. That distinction keeps theoretical cartoons honest when they sit beside evaluated data tables and modern many-body calculations. Along the course arc, write one sentence linking this chapter backward and one forward: the discipline of those two sentences is as important as any single formula. Examination problems rarely ask for a literature review, but they do ask for the scale argument that literature reviews presuppose.

Synthesis problem

Write a half-page note that (i) restates the chapter's central scale or selection rule in one equation, (ii) evaluates it numerically for one concrete nucleus or energy, and (iii) names the neighbouring lecture that uses the result. Keep the note free of research-frontier speculation; the goal is fluency with the lecture physics itself. If an assumption fails (Born approximation, spherical mean field, allowed transition, one-dimensional barrier), state the failure mode in a final sentence. That habit converts passive reading into examination-ready understanding and makes the later research-frontier box read as an extension rather than a replacement.

Research frontier: three-nucleon forces, SRG, and student projects

Yukawa's one-pion exchange (Lecture 11) is the longest-range piece of the residual NN force and still sets the scale $\hbar/(m_\pi c) \sim 1.4\,\text{fm}$. Chiral EFT shows that consistent three-nucleon forces appear at a definite order and are required for saturation and for spectra of light nuclei; fitting only two-nucleon data typically misplaces binding beyond $A \sim 3$ [41, 42].

Similarity renormalization group (SRG) and in-medium SRG methods soften interactions for many-body calculations while tracking induced many-body operators that must not be discarded silently [44]. Connecting lattice-QCD low-energy constants to these EFTs is a long-term goal of the field and a central recommendation of community roadmaps [52]. Remaining uncertainties include the chiral-order truncation error of three-nucleon forces in medium-mass nuclei and the regulator dependence of SRG-evolved Hamiltonians when induced four-body forces are omitted.

A practical computational direction is to compare binding energies of $A = 3, 4$ systems with and without an explicit $3N$ term in a published Hamiltonian family. Even a literature-only comparison of tabulated results shows why the pion-exchange intuition of the lecture must be completed by many-body forces before saturation is trustworthy.

Community roadmaps treat that few-body to many-body pipeline as unfinished business rather than a solved derivation.

Student directions. (1) Analytic estimate: evaluate the Yukawa range $\hbar/(m_\pi c)$ numerically. (2) Conceptual note: explain why a pure two-nucleon force fitted to NN scattering underbinds or overbinds nuclei beyond $A \sim 3$ without $3N$ forces. (3) Methods sketch: outline an SRG flow that decouples high-momentum modes [44].

1.9 Problems with solutions

Problem 11.1 — Pion mass from range

If the nuclear force range is $R = 1.4\,\mathrm{fm}$ and $\hbar c = 197\,\mathrm{MeV\,fm}$, estimate the mediator mass $mc^2 \approx \hbar c/R$. Compare with $m_\pi c^2 \approx 140\,\mathrm{MeV}$.

Solution 11.1

The Compton relation $R = \hbar/(mc)$ rearranges to

$$mc^2 = \frac{\hbar c}{R} \approx \frac{197\,\mathrm{MeV\,fm}}{1.4\,\mathrm{fm}} \approx 141\,\mathrm{MeV},$$

in excellent agreement with $m_\pi c^2 \approx 140\,\mathrm{MeV}$ [9, 14].

Problem 11.2 — Photon vs pion range

Why does electromagnetism have infinite range while the residual strong force does not, in the exchange picture?

Solution 11.2

The photon is massless, so Δt can be arbitrarily long and $R \sim c\Delta t$ unbounded. The pion is massive, so $\Delta t \lesssim \hbar/(m_\pi c^2)$ and $R \sim \hbar/(m_\pi c)$ is a few fm [5, 9].

Problem 11.3 — Which pions in *pp* vs *np*?

Which pion species can mediate pp scattering if both protons remain protons? Which can mediate np?

Solution 11.3

pp: only π^0 (charged emission would turn a proton into a neutron). np: π^0, π^+, and π^- all allowed via charge-exchange and neutral channels [14].

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